‘Why the professor must be a stimulating teacher’

Towards a new paradigm of teaching mathematics at University level

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1. Introduction

Mathematics at the University level is a complex field to explore. The diversity of institutions and social and cultural contexts, the variety of curricula and courses, the reforms taking place at present, etc., may induce us to believe that perhaps it makes no sense to talk about general or common aspects of our academic activities. But after many years of observing our own profession, of visiting so many places around the world and interacting with so many colleagues I have identified some problems and some challenges that may be of interest for mathematicians who love mathematics and love teaching.

The aim of this presentation is to share some critical thoughts and to point out some constructive ideas on the educational goals of teaching mathematics at the university level.

2. Some critical views on existing myths and practices in university teaching of mathematics

In this section I would like to unmask some very general existing ‘myths’ and practices in the teaching of mathematics at the undergraduate level, which have a negative influence on the quality of mathematics teaching:

The researchers-always-make-good-teachers myth

This university myth says that 'researchers are *ipso facto* good teachers… therefore the key criteria for selection and promotion must be high quality research'. Following [1] we quote the statement that:

> *'The standard track assumes a progression through graduate education culminating in a doctorate and followed by a career of teaching and the creation of new mathematics… this is the track which most university faculty members best understand… after all, it is probably the track which they themselves followed'.*

This myth calls for a number of observations.

(i) Sound knowledge does not necessarily mean active research;

(ii) The majority of mathematics courses do not include advanced results reached in recent decades;

(iii) Research takes place in thousands of different specialties, most of it in very narrow fields, and lines of research are often a matter of free choice and quite unrelated to teaching.

(iv) Unfortunately, research criteria are closely related to the Department's interests and rarely include research into mathematics education.

Let us remember here the critical words expressed in [2]:

> *'The mania for research has produced an invidious system of academic promotion, perversion of undergraduate education, and contempt for and flight from teaching'.*

While for graduate, doctoral and post-doctoral teaching activities there is no doubt that only the most up-to-date and active researchers can introduce students to the latest results, techniques and trends, this does not hold true for most undergraduate programs.

The self-made-teacher tradition

This is another standard mathematical myth and is based upon the claim that excellence in university teaching does not require any specific training - it is just a matter of accumulated experience, clear presentation skills and a sound knowledge of the subject. This approach leaves room for a lot of creative freedom but at the same time it can lead to quite a lot of anxiety, especially for inexperienced young teachers, who will in general try to reproduce the models that they have been exposed to during their own education. This myth does not make provision for students who are exposed to various styles of teaching simultaneously and it also avoids the issue of critical input from colleagues as well as the positive training that one would expect from the institutions involved. As Carol Jago said in the article 'Time to ask for whom the bell tolls' [3]:

> 'a thousand teachers making tiny little discoveries in their own classroom will never create systemic change'.

Clearly, teaching may benefit from training and this must be a compulsory activity for those who want to teach.

Context-free universal content

This idea justifies the contents of many courses as 'basic skills and results which must be learned by everyone taking the course'. This myth generated classic courses which were given to almost everyone entering science or technological university studies. It is taken for granted that some elements of linear algebra, calculus, differential equations, discrete mathematics, probability, statistics, etc. constitute the 'core' curriculum of university mathematics. In particular this myth justifies the concept that teaching is context-free, i.e., independent of personal interests, of specific professional training, of cultural environment, of social circumstances, etc. While this situation makes for a more flexible teaching organization (anyone can teach anything), it sacrifices students' interest and kills interdisciplinary approaches. This led to wide and even universal sales for some textbooks. We, however, believe that contents must be related to interest, special needs, context, etc.

Deductive organization

In this case, 'teaching' is thought to be assimilated thanks to representations of deductive thinking. Topics are presented linearly, definitions-theorems-proofs are sequentially stated in their most general form. In particular this presentation leads to the need for constant proofs (the more formal the better) and leaves little room for discussion or historical remarks… 'How?' becomes more important than 'why?'. Is deduction more important than induction? Is formal reasoning more important than plausible thinking? Clearly, deduction is only one component of mathematical thinking.

The top-down approach

This approach holds that by teaching mathematical topics in their most general form, students will be able to deal with any particular case, any example, any application. This gets rid of the problem of real data and the main elements of mathematics modeling. Learning is a bottom-up process, so teaching top-down is not an effective way of helping learners.

The perfect-theory presentation

Mathematics courses present positive results, solved problems, bonified models… students become convinced that mathematics is almost complete, that theorem proving is just a deductive game, that errors, false trials, zig-zag arguments etc., which play such a crucial role in human life, have no place in the mathematical world. Unfortunately, in some ways many textbooks have inherited the cold research-journal style. This style of presentation kidnaps the 'human nature' of mathematical discoveries, the mistakes that were made, the difficulties, the need for simplifications, etc. In some cases (e.g. statistics) this gives the false idea that the 'real subject' is 'the mathematical model', when we know that mathematics may be a powerful tool but it needs to be used in combination with other disciplines or techniques. In addition, we are presented with the paradox that very often this perfect presentation implies only an instrumental understanding instead of a relational understanding. This perfect-theory presentation turns a living discipline into a dead garden.

The 'master class'/formal lecture paradigm

Teaching has frequently been oriented towards 'communicating' mathematical knowledge. Typically, a class for undergraduates would consist of [4] large group of students sitting, listening and writing in a classroom where a professor delivers several hours per week of spoken-written presentation before a blackboard. After the lectures, students are supposed to study the delivered contents by reading notes, the textbook and by solving *ad hoc* exercises proposed for each chapter-talk. This reduces 'teaching' to lecturing, and 'learning' to an individual after-class activity of assimilating results and practicing techniques. In particular, as noted by [5], students spend a lot of time inefficiently or unproductively:

> *'…a considerable part of the time is devoted to the transference from the notes of the lecturer to the notepads of the students of relatively straightforward factual material'*

While 'master classes'/formal lectures are fine when truly 'masterly', they could nevertheless be combined with other techniques of communicating and working.

The mature students myth

At the freshman level, this myth assumes that during the few weeks between high school and university registration, students have grown in such a way that their integration into the new university atmosphere does not require any special attention. In particular, students going into scientific or technical courses are assumed to be already motivated and aware of the relevance of mathematics to their training, and students going into other studies are assumed to constitute a low-interest class. The diversity of backgrounds is often ignored. The high school curriculum may often be unknown. Clearly, the transition from secondary schools to universities needs special attention.

The routine individual-written assessment

This presents the final test, or a written exam mixing questions and exercises, as an ideal method of marking, i.e., of gauging how well students master the contents delivered in lectures. The method focuses on individual preparation and rarely opens doors to project work, group activities, open questions, etc. In its most rigorous form, this assessment is reduced to a final exam to be marked and rarely integrates other activities or information attained during the course on student's progress. More flexible assessment resources should be considered.

The non-emotional audience

This tries to present students enrolled in a course as an audience at a movie show or a theater. The main goal 'for all' is simply mathematics. Individual problems, emotional difficulties, personality features do not belong to the teaching and learning of mathematics. Tuition is for solving technical doubts or clarifying previous lectures. Outside the classroom or the scheduled office hours there is no place for further human interaction. The university walls keep human nature out. To sum up, let me quote [6]:

> *'I don't think that it is healthy for a mathematics teacher to worry about math anxiety. Your job is to teach mathematics. Go do it'.*

That's a terrible mistake. The 'audience' is a group of people in which each individual needs attention.

We, as mathematics educators working at university level, need to destroy the above myths, practices and considerations by taking some positive steps towards another way of teaching.

3. Towards a new paradigm of teaching mathematics at university level

In this section we will identify some changes to be considered, some questions which need to be faced urgently and some goals for our future as mathematicians and mathematics educators.

There is a need to redefine mathematical research as a university activity, combining it with a soundly based teaching excellence

The critical pressure of research has evolved into a crazy rolling snowball: publishing as many papers as possible, going into citation and impact indices, attending an increasing number of congresses… It is time to sit down and think about what the main goals of universities today are. It is just possible that good teaching, fine multimedia and educational materials, virtual projects, community work, etc. are becoming more relevant to administrators and society than subscriptions to journals, abstract announcements and department reports. This does not mean a change from the research-realm to the teaching-paradigm. The 'either-research-or-teaching' polarity is false. With a little wisdom both activities can be (and should be) combined. Research also means writing expository papers, critiques of trends, historical perspectives, good texts, analyses of pedagogical materials, improvement of proofs, suggestions as to new approaches or interdisciplinary applications. Institutions and authorities should recognize and stimulate scholarship and research. And there is no need to say that the creation of exclusive research institutions is to be welcomed. But universities cannot close their eyes to their teaching ends. It is not just a question of achieving one annual award or medal for academic distinction but rather it is a matter of continuously controlling and stimulating the quality of education. Good teaching is [7]: 'building understanding, communicating, engaging, problem solving, nurturing and organizing for learning', a complete task that merits special attention and preparation.

Research into mathematics education at tertiary level may be itself an interesting field of research and may give rise to useful results for all teachers for application to their teaching

Research into mathematics education is a growing scientific discipline (even if Saunders Mac Lane does not approve). Nowadays it involves many researchers focusing on a wide range of topics and levels. However, there is clearly still a rich agenda for research on teaching and learning problems at university level. It would be marvelous if in the years to come this university research attracted well qualified mathematics specialists. If institutions wish or need to pay more attention to their educational goals, then mathematics education may - or indeed is certain to - play an increasingly important role in people's vitae@. Though non-educational research has been a priority in people's careers until now, it could well be healthier if future mathematics specialists combined research with more educational aims.

Moreover, research into mathematics education gives rise to useful results which should be disseminated and used, so that all mathematics teaching staff may benefit from an up-to-date knowledge of this field [8].

All mathematics teachers can benefit from efficient training

While the training of teachers for non-university levels is a well-acknowledged necessity, much has yet to be done at tertiary level. There is much to explore here. While young people entering Departments may benefit greatly from the most experienced instructors, this training needs to be institutionalized and rewarded. Possibly, distance learning training courses would help to solve this problem, and they could include such materials as interactive CD-ROMS with case studies or video taped classes, audiovisual resources, examples of model teaching procedures; alternative skills in teaching, etc.). The development of specific societies or associations (or working groups within the existing ones) would also facilitate this training.

Mathematical contents of courses need continuous reform, renewal and a close link between what students learn and what they will need in their near future

In short, context is important for what we teach, motivation is essential, modes of thought are key concepts… Contents are not ends in themselves but means to ensure efficient knowledge. Examples, applications, historical backgrounds, modeling processes, the results that have been achieved in recent years… all these are or must be *'spiritus movens'* of the curricula. Let us remember here some words of Sol Garfunkel on the 'Principles and Practice of Mathematics':

> *'The central importance of mathematics in our technologically complex world is undeniable, and the possibilities for new applications are almost endless. But at the undergraduate level, little of this excitement is being conveyed to our students. Currently, attention is being focused on reforming calculus, the traditional gateway course into the undergraduate curriculum. No one is questioning the importance and beauty of continuous mathematics. However, reformed or not, calculus is one branch*

(and a highly technical one) of a very rich subject. We know the breadth and richness of our subject; how, then, do we expect the students who are starting their study to gain these insights?…

Our proposed new start or gateway into the college mathematics curriculum is only a revolutionary idea for our discipline; other disciplines have had such courses in place for years…

We asked ourselves a simple question: in designing the first undergraduate course for math and science majors, what should such a course look like?…

The course content stresses the breadth of mathematics, discrete and continuous, probabilistic as well as deterministic, algorithmic and conceptual. We emphasize applications that are both real and immediate. And the text includes topics from modern mathematics that are currently homeless in the undergraduate curriculum'.

Innovative teaching approaches will have a positive effect on learning processes and may affect the mathematical contents, the dynamics of teaching and some basic educational problems such as developing sound assessment methods.

We would like to distinguish three levels of innovative teaching, corresponding to tools to be used, new pedagogical strategies and the issue of assessment.

(i) *Innovative technological tools*

Beyond chalk we have at hand software packages (graphical, numerical, symbolical…), video tapes and audio-cassettes, CD-ROMs, CD-I… and a wide variety of technological devices: computers, networks, televisions, radios, phone-fax-modems, overhead-computer projectors, etc. These tools can be incorporated into multimedia classrooms, computer labs and virtual campuses, and allow us to combine synchronic and asynchronic communications, presentations and interaction.

Visualizing statistics data with Minitab; analyzing polynomial approximations or splines by means of Mathematica, Derive, Maple, Math Lab or whatever; visiting the polyhedra sites on the World Wide Web; showing COMAP videos on real mathematical modeling processes; working on historical mathematics on Internet data bases; simulating random processes on a work station; navigating on an interactive mathematical CD-ROM… all these examples are non-traditional tools and can be applied perfectly well to university topics.

(ii) *Innovative pedagogical strategies*

Problem solving, Internet forums, on-line tutorials, homework on the net, guided reading, simulation experiences, workshops, ateliers, group working, cooperative learning… a rich collection of dynamic learning strategies may be combined. Some of them assign an unconventional role to teachers but all of them contribute to *stimulating motivation, interest and active participation.* A surprising fact, proved in research into mathematics education, is that people learn more by talking and writing than by listening. Some of these pedagogical strategies come either from high school level or from distance/open learning environments, but they have received scant attention at the university level and far more development is needed.

(iii) *Innovative modes of assessment*

Open problems, project work, group projects, portfolios, critical reading… these, and other examples may lead to a more interesting approach towards a positive assessment method in which students can show what they have achieved, their capacities, their success as learners, etc. Continuous assessment is more demanding than final grading and requires small groups, but it is certainly an excellent way to produce assessment tasks that help the students to realize their real level of achievement. This means assessment must be considered as an informative and positive tool for individuals, and not as the traditional final diagnosis.

Mathematics courses need a complete review and clarification of what kind of professionals are trained

Mathematics as a service subject may be controversial and may need special consensus between mathematics specialists and other professionals. But the present undergraduate mathematics course in many places are completely out of this world. These courses train 'mathematicians', i.e., people who know mathematics', but they are often undefined in terms of the professionals they wish to offer to the market. The usual paradox is that maths majors may be quite well prepared for entry into pure or applied research where few jobs are available. But most people go into teaching jobs, commercial divisions, computer companies, etc. positions for which they have not been trained. It would be desirable for mathematics courses to attract more brilliant people (many nowadays go into other technical studies) and for some elements of specialization to be introduced: for teaching, finance, engineering… The 'pure or applied' dichotomy is just a false opposition. Perhaps it is time to break mathematics courses into several separate fields of studies with specific goals. The departmental structure needs a review.

Mathematics as a service subject must teach students a certain body of knowledge, techniques and principles which will be relevant to their training as professionals, and it must show students how to acquire more knowledge and how to use it

In [9] one can find interesting views on mathematics as a service subject. Let me quote here just two of the stronger statements that originated from the ICMI Study: it would be desirable that service mathematics courses enabled students to acquire a range of essential knowledge, skills and modes of mathematical thought; each professional activity demands a particular mathematical literacy so mathematics courses must include applications, examples, modeling processes, etc. which motivate student.

Mathematicians must teach these courses but they need to be ready to prepare themselves for the job. Very often, mathematics has played a selective role in undergraduate studies. This is not true anymore and time devoted to mathematics has suffered a dramatic cut. It is therefore essential to redefine service subject courses to include the most basic training in useful mathematics. Let me quote here the old claim of M. Kline:

> *'…the only generally understood fact about the subject is that it is understandable… professors are content to offer courses that reflect their own values at the expense of students' needs and interests.'*

If we do not want to be seen this way, then an interdisciplinary approach is called for.

Technology is a keystone for innovative global teaching of mathematics at universities in a worldwide situation

We mentioned earlier the impact of technology on the content and dynamics of teaching standard mathematics courses (calculus, algebra, discrete mathematics, statistics, analysis, etc.). Here we would like to point out an area awaiting exploration: technology for distance learning or open programs.

We have a fascinating agenda in front of us: what will the ‘classroom’ be like in the near future? How will we use interactive materials and on-line tuition and e-mail home-work? how will we be teaching students from different continents without moving from where we are? In a global world the walls of our classroom do not exist. Distance education was once the reserve of a limited number of institutions but now all universities are entering the area of high-tech developments to offer distance learning courses. Within a few years, all of us will be competitors on a worldwide scale. What consequences will this worldwide competition have? Virtual campuses, satellite courses, non-synchronic teaching… all this demands deep research and innovative approaches.

Learning is not only preparation for the near future but a lifetime project: This new demand opens major questions for mathematics education in universities

There are more and more people going into higher education. But we also need to face the question of what we are going to offer in continuous education. Distance learning materials, teleteaching, short courses… what kind of contents are we going to offer and how? The increasing changes of jobs and technologies in professional life open up new demands [10]. This is a challenging problem to be solved. Short specific courses based upon the new technologies as delivery systems may be a good place to start. Here again, the appropriate mathematics teaching for adults will be necessary.

Teaching mathematics at university level should be an enjoyable human experience in which professors share with students the discovery of a new mathematical world as well as their development as persons

Teachers and students are, before all else, human beings with brains and hearts. They have the opportunity to share experiences and enthusiasm, to learn from each other. Motivation, attitudes, feelings, guidance, discovery… these are the words that must fill our teaching job. We have the opportunity to transmit our passion for mathematics and this is something that all students may remember. We do not need to be actors or actresses in a mathematical play but rather the partners in a rewarding journey.

4. Time for action?

It is time for action. It is time to promote the teaching vocation of mathematicians. It is time to redefine the equilibrium between research and teaching. I am convinced that it is better to undertake this reform from inside than to wait to have decisions imposed from outside. Today's universities cannot afford to have ivory towers. But we know from past experience that all educational changes need time, effort and consensus. It will take a lot of work to do away with the old traditions and to gradually build up a new approach. But the clock is running.

New social demands from the job market, new needs in training, life-long learning, demographic explosions, technological developments, and above all our love for mathematics and our passion for sharing this enthusiasm with others… all this must shape our drive to find our challenges and our answers for the future. Their future is ours!

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