

TOO MUCH IS NOT ENOUGH
Teaching maths through useful applications
with local and global perspectives

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Abstract

Their future is our future

An initial thought...

Education is for the majority of people the only key for their future. Mathematics is in that sense a major tooth to allow this key to turn in the keyhole of our society.

When learning mathematics was a minority activity, mathematics could be "universal". Now that we want to reach "all" people "everywhere" mathematics need to play a subtle game of giving to anyone as much opportunities as possible.

The world has changed; the perspectives, values and aims of our students have changed... so it is time for us to make some crucial changes in mathematics education.

Local citizens with global perspectives

At compulsory level we prepare future citizens in a very concrete society. Our math teaching may benefit from the local characteristics and it is our goal to prepare students to be critical citizens and good professionals for their contexts.

On the other hand the increasing phenomenon of globalization needs to be faced and clarified, since the term "globalization" hides and mixes various meanings in political terms, in economical settings, in immigration processes, etc.

From our perspective as mathematics educators we need to take into account, mainly, the following dimensions of globalization:

- **Global common problems**

We can develop understanding of these problems (large scale economies, ecological problems,...) and to contribute to quantitative critical views.

- **Global information networks**

Internet, international TV-channels, radios, news agencies,... give an infinite set of possibilities for practising languages or for dealing with interesting cultural contexts, i.e., we have opportunities in math classrooms for learning how to reach informations, for comparing them, etc.

- **Global markets**

We share products from everywhere so there is a need to recognize qualities, offers, discounts, to require certificates, etc.

- **Global travelling**

Most people will have opportunities to visit many places so there is a need to plan trips, to evaluate costs, to know how to use maps, to exchange money, etc.

- **Global immigration of individuals**

The phenomenon of immigration from poor countries to richer ones will go on, so this opens in many classrooms the possibilities of integrating many cultural contributions but giving priority to the local culture.

Our chief concern in this lecture is to see how math teaching through applications can be sensitive to this local and global perspectives.

If learning is the question... quantitative literacy is the answer

In (ILSS,2000) the International Life Skills Survey we can find a very clear and comprehensive definition of **quantitative literacy**:

"An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need

in order to engage effectively in quantitative situations arising in life and work",

and, in similar terms, the Programme for International Student Assessment (PISA, 2000) defines *mathematics literacy* by means of the following terms:

"An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen".

Recently, Lynn Arthur Steen (see (Steen, 2001 p. 8-9) has identified a very complete list of elements which are characteristic components of **quantitative literacy**:

• Confidence with Mathematics	• Mathematics in context
• Cultural Appreciation	• Number sense
• Interpreting Data	• Practical Skills
• Logical Thinking	• Symbol Sense
• Making Decisions	• Prerequisite knowledge

If we accept as starting point that compulsory education needs to focus on quantitative literacy then it is easy to see that *dealing with applications and modeling skills* is an ideal way to approach our teaching/learning goals.

For example a consequence of quantitative literacy must be to develop a *mathematical common sense*, i.e. an adequate mixture of number sense and practical skills. Let me recall in this point that "For all practical purposes" is not only a beautiful text made by COMAP where most issues are presented in a practical way but "For all practical purposes" is a sentence appearing in the 1956 text of C. Stanley Ogilvy (Through the Mathescope, Oxford, University Press, London). In this text a student asks to a professor for the "meaning" of the expression "For all practical purposes" and the answer of the professor is the following:

"Suppose all the young men in this class were to line up on one side of the room, and all the young ladies on the other. At a given signal the two lines move toward each other, halving the distance between them. At a

second signal, they move forward again, halving the remaining distance; and so on at each succeeding signal. Theoretically, the boys would never reach the girls, but actually, after a relatively small number of moves, they would be close enough for all practical purposes".

But beyond suggestive descriptions like this one, it is necessary to face a very serious issue: the mathematical common sense and the skills to deal with different contexts need to be worked in the classroom and not to be killed by the formal-abstract development.

If quantitative literacy is the answer... applications are the solution

Through this lecture we will defend that realistic teaching is an appropriate way towards quantitative literacy training. Motivating by means of useful applications, working on modeling, solving problems... that's the way we consider ideal for our mathematics teaching at all levels.

A remarkable consequence of teaching "via applications" is that the classical way to deliver lectures needs to be changed. Teaching with applications means today to stop the "talk & chalk" method, to stop using an old textbook and to offer a very lively guiding program, based upon various informative sources, with revised teaching notes and opening new windows to appreciate the context of the students and their creativity as individuals and as a group. Under this assumption, the problem is to have some criteria "to choose" applications that will have teaching/learning value.

The word application has been degenerating for a long time. Since any logical consequence or corollary from a well established proposition is the result of "applying" the previous finding... any mathematical discovery could be presented as an "application". While this has a clear meaning in the deseperated search for grants, it brings a confusing situation in the classrooms.

Let me mention here some interesting research results in mathematics education made by the educational psychologist Lieven Verschaffel. It has been shown that after some years of traditional-formal mathematical schooling, students experience the so-called "suspension of sense making in mathematical modeling and problem solving", i.e., students lost common sense and ingenuity when dealing with real life issues in the classroom.

In an evaluation of knowledges and skills, made in Catalonia for 10 years old children it was detected, e.g., that they had hard problems when reading scales in thermometers, numerical values in vertical axis or train time-tables. The confusing situation was not due to the numbers, was due to the presence of images of real objects!.

So we will focus only on *real applications*, based on *real data*, on *objects* and *instruments*, on *daly life* situations, on *frequent* or *recent events*. Along these lines, the american educational projects "Modelling our world" and "Maths in context" are very fine examples of how textbooks can facilitate the realistic approach.

Local and global issues in choosing applications

At the compulsory level it would be convenient to find interesting applications to be considered in the classroom. These applications may play a double role. One is to provide motivations for learning mathematics or to practice knowledges already given. The other is to prepare our future citizens in "their" context.

Next table sums up (in random order) various aspects that may influence our choice of applications:

LOCAL ISSUES	GLOBAL ISSUES
1. The place of the school	6. The global world
2. The social and cultural characteristics	7. The technology available
3. The time's factors	8. The critical views
4. The level of learners	9. The appropriate topics
5. The skills of teachers	10. The previsible future

To clarify this we are going to make some remarks on this ten aspects.

- **The place of the school**

In what point of the Blue Planet are we teaching? Are we in a big city? Are we in a small village? Are we in a developed country?... We must be sensitive to the place where we are. Some environments are rich in motivating contexts,

others are not. We may take advantage of the place or, otherwise, we may need to supply "additional motivation".

Do we have factories to visit? Are good measures available in the zone? Do we have singular buildings? How is public transportation organised? How is pollution measures?... If the answers are "yes" then we will have at hand interesting places to generate mathematical activities. Otherwise by means of Internet, books, pictures, etc. we may "bring" to the classroom fine inputs.

- **The social and cultural characteristics**

In the place where we are, in addition of geographical or architectural possibilities, there are social demands, social issues to be faced, cultural activities, etc.

We need to take into account as much as possible this motivating situations.

Dramatical procedures, working conditions, retirements plans, economical indices, inflation, theater plays, television, book reading, local dances, music, musical instruments, cuisine... all social and cultural realities may have some mathematical interest.

- **The time's factors**

In which decade or century are we teaching? Which things happened recently? Which are today's news? Which are the immediate events?... To connect mathematics and time-reality is a must!

Many maths examples presented in frozen textbooks are useless because the applications which are explained are already overcome. While "old" applications are nice for making historial considerations we can't take them as actual practices. Problem with triangles realted to land's measures by means of topographical instruments are useful. Measures with knots and shadows are useless.

- **The level of learners**

Clearly the level of learners will orientate us on which choice to make concerning applications. While a tender fiction tale on numeracy may be

appropriate in kindergarden, there is no way to explain it in a High School. Each generation of students has topics which are relevant for them... and we want to interest them.

Many times in recreational mathematics problems are presented in a fiction-real context which insinuates that the result to discover will be a crucial issue for one's life. Crossing rivers, climbing castle, covering chess boards with tetraminoes... who is doing these things today? Useless maths can't become useful even if they are presented in a funny way.

"Cooked" examples to illustrate some mathematical concepts or results are related to situations which are not interesting for the students or even for the teachers. Let us recall the old problem "If 5 workers in a building will end the work in 3 weeks, when the building will be finished if 25 workers are assigned?".

Applications (and especially research activities on them) are ideal items to induce cooperative work or team work. Good assessment e.g. in making projects needs to mark individual and cooperative aspects. To prepare for cooperative work is a crucial goal under today's circumstances.

- **The skills of teachers**

The most beautiful and motivating application of the world may become a disaster in the hands of an insecure teacher. The confidence of teachers in the examples they choose and their skills to make mathematics visible from them, are factors to be seriously considered.

To deal with applications, modelling procedures and problem solving techniques require extra efforts. This implies a need for training teachers to be prepared for making these activities possible and fruitful.

- **The global world**

Our world has today a global dimension. We have many problems and things in common. We may discuss about these issues (ecological disasters, increasing heat, economical dependence,...) but we need to prepare our students to deal with this international setting. For some of them the work market is (or will be) global, but most of them will have opportunities to know various places.

One may distinguish anyway between "applications" which are being "made" by many people from those which are being "used" by many but made by few. CD's is a typical example. The NASA projects are another example. "Interesting" does not means "useful".

- **The technology available**

The changing technological devices (hardware, software, photos,...) offer a great deal of learning possibilities. We are refering not only to the case of computers and scientific calculators but to all kinds of technologies, specially those allowing communications and those opening creativity.

- **The critical views**

So many informations, so many commercial, so many TV channels, so many magazines,... all these global products and culture that we share imply the need (more than ever before) to develop in the classrooms tools for critical views. Mathematics (e.g. through data analysis, visual displays, statistics,...) may be a key arm to attack quantitative issues.

- **The appropriate topics**

Concrete applications may play an special role in treating some mathematical topics. Thus while transport problems may benefit from graph theory, architectural designs may be excellent examples for geometrical modeling.

- **The previsible future**

Since we want to prepare citizens for "their" future some tentative prospective views must be considered. Which issues will be important in the next decade? How can we project today's data to have some intuition on further developments?

Following the spirit of the above discussion we present in tables 1 and 2 examples of 50 applications corresponding to the ten aspects we just described.

EXAMPLES OF LOCAL APPLICATIONS FOR TEACHING PURPOSES	
1	<ul style="list-style-type: none"> • Geometrical characteristics and measures of the schools' building • Distances from school, times to reach the school • Geographical coordinates of the center of the school • Sources of informations in the school • Food in the school and costs
2	<ul style="list-style-type: none"> • Measures and proportions in popular dishes • Geometry of singular buildings in town • Mathematics in folk dances • Different scales in local charts • Statistical study of minorities in the local society
3	<ul style="list-style-type: none"> • History of calendars. The local holidays • Mathematics and sports. Olympiads. World's records • Art exhibits in town • Mathematics in newspapers and magazines • Mathematics in consume issues. Indices.
4	<ul style="list-style-type: none"> • Numbers and classical tales • Numbers in popular sayings • Mathematics and music • Alcohol rates and driving: waiting times • The cost of having a moto-cycle
5	<ul style="list-style-type: none"> • Organize visits to industries using mathematical principles • Select projects to be used for team assessment • Design a math lab for the classroom • Organize maths activities for a weekend workshop • Collect teaching materials from daly life objects

Table 1

EXAMPLES OF GLOBAL APPLICATIONS FOR TEACHING PURPOSES	
6	<ul style="list-style-type: none"> • Demographical issues: perspectives and extrapolations • Insurances in travelling: accidents' frequencies,... • Mathematics in democracy • Solidarity funds • Ecological problems: the Kyoto agreement
7	<ul style="list-style-type: none"> • Mathematics and traffic (cars, roads, gas,...) • Locations in the planet (GMS system) • Air traffic control: the keys of not crashing • Digital images as messages • Implications of air condition in housing
8	<ul style="list-style-type: none"> • Statistics on prices: imports and exports • Visual display from everywhere • A visit to a car's factory: sequential working • A visit to food's factory: quality control • Statistics about political elections: pools and votes
9	<ul style="list-style-type: none"> • Codes, phones and messages • Mathematics and genomics • Art: paintings, sculptures, buildings • Geometry in theme parks • Fair division: geometry and equity
10	<ul style="list-style-type: none"> • Mathematics and information: CD versus DVD... • Security of data in Internet • Virtual museums about Science and Technology • Airplanes: speeds and capacities • Geometry and virtual 3D-images

Table 2

Hands on teaching materials

Let me remember that some years ago, in April 1957, the 11th Meeting of the CIEAEM took place in Madrid, in the public school Instituto de San Isidro. The "spiritus movens" of the meeting was Pere Puig-Adam and the main topic was "Teaching materials". Pere Puig-Adam had a strong influence in mathematics education in Spain and the publication in Spanish of the CIEAEM proceedings (Gattegno et al., 1967) became an interesting source of ideas. This text benefited from the contributions of C. Gattegno, E. Castelnuovo, J.C. Nicolet, J.J. Fletcher, L. Motard, L. Campedelli, A. Bignenet, J.W. Peskett and P. Puig-Adam and most of the ideas which were described in this text still have actuality.

I would like to bring back the spirit of this old CIEAEM meeting and to defend **the need of producing good teaching materials**. With the exception of some elementary geometrical shapes, the market does not produce materials for selling. On the other hand it is frequently argued that hands-on material may be helpful only at the very elementary levels.

If we want to show applications and modeling procedures we can find in our home-made materials great opportunities **to bring "real" objects to the class and to provoke an experimental research approach by modeling by means of specific materials**.

Most materials can help to promote an interdisciplinary dialogue, may help to visualise properties in a fine way... and may increase interest in the formulated problems at all levels.

Example: Beware with the steps of a stair!

This is an example to be studied with materials at scale 1:1 and which has universal value: all humans need stairs which are easy to climb and almost human beings use shoes.

Stairs are important objects. Measure them! (e.g. using electronic measures). The ideal steps have two important measures: H (height) and D (depth) related by the affine equation $2H+D=63\text{cm}$. the inclination $\tan A = H:D$ is also interesting. Which are upper and lower bounds for H, D and A? When is it convenient to have a ramp and not a stair? In vertical stair (like in submarines)

you face the steps to go down, but in normal stairs you come down the other way around: when is it better to face the steps?

Example: Modeling in the rain... with umbrellas

Today's umbrellas are sophisticated folding structures but they share with the old ones a beautiful geometrical fact: the regular 8-gon of their extrem points. When you take out the stuff from the top and observe the moving octagonal pyramid you may discover how several parallelograms change angles (and areas) but keep perimeters. All these parallelograms determine a moving bipyramid... and if you joint two extreme points of the 8-gon you obtain a moving 7-gon which may be used to mark in a given circle the 7-gon (one of the impossible solutions with rule and compass!).

It is interesting to collect umbrellas from the world. Is their top surface related to the rain they support? Which is the difference between a chinese and a japanese umbrella?

Example: Kitchen's tools

In any kitchen of the world one can find a beautiful collection of useful utensils. One can formulate good questions about them: does it have the ideal shape? is its shape optimal for its function? can they be used by left-handed people?

We recommend you to deal with: pots and their covers, scissors, measuring pots with scales, spoon to separate the yolk of an egg from its white, tea-tools, cylinders for cleaning gardening's skins, folding, glasses, sticks,... a complete international collection.

...but do not forget the key point!

To end this presentation let us make clear something which is crucial. All kinds of applications may be helpful to make visible mathematical dimensions of our daily life in the Planet. But you need to remember that among all the things you bring to the classroom there is one which is essential. There may be no chalk and there may be no materials, but at least one thing can not be forgotten: YOUR ENTHUSIASM AND YOUR ILLUSION in mathematics, in the students and in your job.

Thanks for bringing all these virtues everyday to the classes!... Too much is not enough!

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