200243 - LF - Logic and Foundations

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 726 - MA II - Department of Applied Mathematics II
Academic year: 2013
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Optional)
ECTS credits: 6
Teaching languages: Catalan

Teaching staff

Coordinator: RAIMON ELGUETA MONTO
Others: RAIMON ELGUETA MONTO - A
        FRANCESC TIÑENA SALVAÑA - A

Degree competences to which the subject contributes

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking into account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

General:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.

7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.

8. CG-3. Have the ability to define new mathematical objects in terms of others already known and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Teaching methodology

(Section not available)
### Learning objectives of the subject

(Section not available)

### Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group: 30h</th>
<th>20.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Hours small group: 30h</td>
<td>20.00%</td>
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<tr>
<td></td>
<td>Guided activities: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Self study: 90h</td>
<td>60.00%</td>
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Content

1. Introduction: The Hilbert program for the foundations of mathematics
   Learning time: 1h
   Theory classes: 1h

2. Background: The transformation of mathematics in the 19th century
   Learning time: 21h
   Theory classes: 11h
   Guided activities: 3h
   Self study: 7h

   Description:
   A. Mathematical objects.
   Preliminaries: Numbers and magnitudes in Greek mathematics. The construction of the reals. Transformation of
   the notion of integer. The extension of finite numbers: transfinite numbers and set theory. The ontological and
   methodological debate: paradoxes of the infinite.

   B. The evolution of the method.
   Introduction: certainty of mathematical knowledge, the use of the axioms and language. The axiomatic method:
   from Greek deductive mathematics to Hilbert's conception. The emergence of symbolic logic: from sillogistics to
   the symbolic calculi of the end of 19th century.

3. Hilbert's program: context and development
   Learning time: 70h
   Theory classes: 25h
   Guided activities: 10h
   Self study: 35h

   Description:
   A. Formulation: 1900-1921
   Situation in the turn of the century: the crises of foundations. Period 1900-1905: Hilbert's lecture in Paris, the
   first outline of the program in 1905 and Poincaré's critiques. The alternatives: Russell's logicist solution,
   axiomatic proposal by Zermelo and Brouwer and Weyl's intuitionism. Hilbert's reaction: The emergence of first
   order logic and the formulation of the program in 1921.

   B. Contributions: 1921-1936
   The completeness of logic: Gödel's theorem, compactness and Skolem's paradox. The decision problem: Notion
   of algorithm and the indecidability of first order logic. The incompleteness phenomenon and the necessity of a
   reformulation of Hilbert's program. Proofs of consistency: The extension of finitary methods and Gentzen's
   consistency proof for arithmetic.
The course grade (N) is obtained from:
* Delivery of exercises during the course (P) (they consist in a written brief discussion of a theme proposed by the teacher or in solving a problem), and
* A final exam (F).
Then, \[ N = 0.4P + 0.6F. \]

### Bibliography

#### Basic:

#### Complementary: