250122 - GDIFEQDIF - Differential Geometry and Differential Equations

Coordinating unit: 250 - ETSECCPB - Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering
Academic year: 2017
Degree: BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2010). (Teaching unit Compulsory) BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2017). (Teaching unit Compulsory)
ECTS credits: 9

Teaching languages: Spanish, English

Coordinator: MARINO ARROYO BALAGUER, ENRIQUE BENDITO PEREZ

Others: IRENE ARIAS VICENTE, MARINO ARROYO BALAGUER, ENRIQUE BENDITO PEREZ, PEDRO DIEZ MEJIA, ANTONIO HUERTA CEREZUELA

Opening hours

Timetable: To be arranged in the course.

Degree competences to which the subject contributes

Specific:
3050. Ability to provide analytical descriptions of curves and surfaces, calculate their properties and perform differential calculus operations on them; find analytical solutions to complex contour and initial value problems in various dimensions and with simple geometrical conditions enabling an analysis, including a parametric study, to be made of these solutions

Transversal:
588. SUSTAINABILITY AND SOCIAL COMMITMENT - Level 1. Analyzing the world’s situation critically and systemically, while taking an interdisciplinary approach to sustainability and adhering to the principles of sustainable human development. Recognizing the social and environmental implications of a particular professional activity.
591. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
598. EFFECTIVE USE OF INFORMATION RESOURCES - Level 2. Designing and executing a good strategy for advanced searches using specialized information resources, once the various parts of an academic document have been identified and bibliographical references provided. Choosing suitable information based on its relevance and quality.
601. SELF-DIRECTED LEARNING - Level 2: Completing set tasks based on the guidelines set by lecturers. Devoting the time needed to complete each task, including personal contributions and expanding on the recommended information sources.
Teaching methodology

The course consists of 6 hours per week of classroom to classroom (large group).

They are devoted to lectures 2.5 hours largest group, the teacher presents the basic concepts and materials of matter, presents examples and exercising.

It is dedicated 3.5 hours largest group, solving problems with greater interaction with the student. Practical exercises to consolidate the objectives of general and specific learning.

The rest of weekly hours dedicated to evaluations.

Support material is used in the form of detailed teaching plan using the virtual campus ATENEA: content, programming and evaluation activities directed learning and literature.

Learning objectives of the subject

Students will acquire an understanding of differential geometry (including curves and surfaces, as well as integration on manifolds and integral theorems) and partial differential equations of mathematical physics. They will also develop the skills to analyse and solve mathematical problems in engineering that involve these concepts.

Upon completion of the course, students will have acquired the ability to: 1. Relate partial differential equations to engineering problems in continuous media. 2. Program complex solutions using basic software and obtain numerical solutions. 3. Develop analytical solutions to complex multidimensional boundary value and initial value problems with simple geometric conditions that allow an analysis of these solutions, including a parametric study. 4. Carry out an analytical description of curves and surfaces, calculate their properties, and perform differential and integral calculus operations on them.

Basic tools in metric geometry: Ruler-and-compass constructions and demonstrations; Floor plans; Technical drawing; The conic system.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 225h</th>
<th>Theory classes: 32h</th>
<th>14.22%</th>
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<tbody>
<tr>
<td></td>
<td>Practical classes: 30h</td>
<td>13.33%</td>
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<tr>
<td></td>
<td>Laboratory classes: 28h</td>
<td>12.44%</td>
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<td></td>
<td>Guided activities: 9h</td>
<td>4.00%</td>
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<tr>
<td></td>
<td>Self study: 126h</td>
<td>56.00%</td>
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## Content

### CHANGE OF VARIABLES

<table>
<thead>
<tr>
<th>Learning time: 24h</th>
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<tbody>
<tr>
<td>Theory classes: 4h</td>
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<tr>
<td>Practical classes: 6h</td>
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<tr>
<td>Self study: 14h</td>
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**Description:**
To demonstrate the usefulness of the Inverse Function Theorem
Describe the most interesting curvilinear coordinates in the two-and three-dimensional Euclidean space
Consideration of vector spaces, of the same dimension of the object coordinate, in each of its points.
Construction of the bases of vector spaces as tangent vectors to the coordinate curves

**Specific objectives:**
Understand the concept of change of variables respecting the regularity properties
Learn to make the most usual change of variable
Describe vector fields related to coordinate objects
Understanding the Jacobian of curvilinear coordinates as the change of variable matrix for vector fields

### PARAMETRIZED CURVES

<table>
<thead>
<tr>
<th>Learning time: 19h 12m</th>
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<tbody>
<tr>
<td>Theory classes: 4h</td>
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<tr>
<td>Practical classes: 4h</td>
</tr>
<tr>
<td>Self study: 11h 12m</td>
</tr>
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**Description:**
Change of parameter and tangent to curves
Qualitative understanding of regular curves

**Specific objectives:**
Know the different kinematic concepts related to the trajectory of a point in geometric terms
Construct the minimum information necessary to distinguish the curves parameterized except for rigid motion

### GEOMETRY EXAM

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<tbody>
<tr>
<td>Laboratory classes: 6h</td>
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<td>Self study: 8h 23m</td>
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### GEOMETRY OF SURFACES

**Description:**
- Understanding the structure of surfaces and the control of their regularity
- Determine the surfaces and their aspect by the way of to describe
- Diffeomorphisms on surfaces and the First Fundamental Form
- The gaussian curvature and other curvatures in coordinates

**Specific objectives:**
- Distinguish why an application is a surface and evaluate when it has a tangent plane
- Knowing ruled surfaces, surfaces of revolution and level surfaces
- Consider the composition with the parametrizations to extend the differential calculus on the surfaces and introduce the ability to measure on surfaces
- Get the curvatures of any parameterized surface and understand the major characteristics of the surfaces most used

**Learning time:** 19h 12m
- Theory classes: 3h
- Practical classes: 5h
- Self study: 11h 12m

### MANIFOLDS WITH BOUNDARY

**Description:**
- Surface extension of the concept of regular and metric dimension n objects in a space of dimension s environment
- Consider parameterizations of closed intervals and closed pseudo-intervals, and characterize the orientation induced on the boundary of the parameterization

**Specific objectives:**
- Provide a common language to curves, surfaces, and open two-and three-dimensional
- Know the description of a variety of objects such as finite union of pseudo-intervals or for the parameterization of these

**Learning time:** 9h 36m
- Theory classes: 2h
- Practical classes: 2h
- Self study: 5h 36m
### Stokes Theorem

**Description:**
Obtaining the Stokes Theorem on parameterized intervals, by applying the Barrow Rule in each variable. Getting the divergence theorem, Stokes-Ampère and Green in the plane, as elementary application of general Stokes theorem. Solving exercises in which they appear most distinctive resources that facilitate the resolution of a wide range of problems.

**Specific objectives:**
- Understanding the scope of integration of functions and the correspondence between the implementation of integration within the objects or on their boundary.
- Understanding the relationship between divergence, flow and circulation of vector fields on tri and bidimensional objects and their boundaries.
- Learning to solve problems of integration on complex objects, but whose pieces are described by simple expressions that facilitate the realization of calculations on them.

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### Variations of Integrals

**Description:**
Knowledge of the variation of integrals with respect to the time when both subintegral function as the domain of integration depend of the time. Getting some conservation laws, as pure application of the results of integration and basic mechanical concepts in elementary media.

**Specific objectives:**
- Using elementary techniques of integration and implementation of the classic theorems, connect with key elements in the description of continuous media.
- Spending on basic physical laws from a simplified perspective emphasizing techniques of vector calculus.

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### CALCULUS OF VARIATIONS

**Learning time:** 14h 23m  
**Description:**  
Formulation of the problem, necessary condition for extrema, Euler and Euler-Lagrange equations and natural boundary conditions  
Systematically explore the problems posed by the necessary condition for extrema, identifying the characteristics of the different types of equations obtained, self-adjoint problems, problems of one or more variables, eigenvalues, natural boundary conditions or enforced  
**Specific objectives:**  
It is intended to provide a systematic way of expose problems, covering the most relevant part of the problems that arise in the context of differential equations in both one variable as in several variables  
It is intended to come into contact with a number of problems that will gradually expand the horizon of action, from the classic problem of brachistochrone to the equations from the minimization of elastic strain energy

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### MATHEMATICAL PHYSICS EQUATIONS

**Learning time:** 14h 23m  
**Description:**  
From the minimization of quadratic functionals and considering isoperimetric conditions, with functions of one variable, we obtain linear boundary problems and eigenvalues problems for ODE's  
Will be posed classification criteria and the prototypes of elliptic equations, parabolic equations and hyperbolic equations  
**Specific objectives:**  
Aims to provide tools to solve these problems of great interest in applications, the tools previously known consist only in rudiments to address basic problems of initial value  
We present families of problems that can be tackle, heat equations, wave equation, Laplace equation, types of domains and the initial and boundary conditions

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### QUADRATIC FUNCTIONALS

**Learning time:** 14h 23m  
**Description:**  
Estimation and analysis of eigenvalues of Sturm-Liouville problems, and solving boundary value problems including the use of Green's functions  
**Specific objectives:**  
Gain experience in using methods of solving such problems, these techniques remain in force when dealing with problems of partial differential equations by separation of variables
### EQUATIONS EXAM

**Learning time:** 14h 23m  
Laboratory classes: 6h  
Self study: 8h 23m

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### SEPARATION OF VARIABLES

**Learning time:** 19h 12m  
Practical classes: 8h  
Self study: 11h 12m

**Description:**  
We introduce the Fourier method for the one-dimensional diffusion problems, and we extend this method of resolution to any problem that fits with it. This allows us to resolve many issues previously raised whose solutions will be represented in orthonormal bases.

**Specific objectives:**  
Have a tool to obtain effective analytical solutions of problems of interest in many applied areas and also understand its scope, on the other hand it allows to assess the need for other techniques to address problems that cannot be solved with the method.

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### APPROXIMATED SOLUTIONS

**Learning time:** 24h  
Theory classes: 5h  
Practical classes: 5h  
Self study: 14h

**Description:**  
The direct methods consist in considering minimizing sequences and are not easy to systematize, the weak form is to write the equations in integral form and with less regularity requirements and are easy to systematize.

Expose a method to control the coefficients of the existing problems, to obtain the approximate solution in finite-dimensional spaces.

**Specific objectives:**  
Consider alternative methods to the separation of variables for the solution of problems of great interest and expose the rudiments to the weak form as the way for implementation the approximation methods for many problems.  
Access the basic rudiments so that, throughout their studies, they can know the methods of numerical solution of many problems of interest.
Qualification system

There will be four written exams of the subject. Two in Differential Geometry area, G1 and G2, and two in Differential Equations area, E1 and E2.

The exams will have the same value for the final note

The final note will, \( NF = \frac{(G1 + G2 + E1 + E2)}{4} \).

Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation tests will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

These tests must be authorized by the corresponding Head of Studies, at the request of the professor responsible for the course, and will be carried out within the corresponding academic period.

Evaluation in the English Group

The evaluation will consist of three elements. 30% of the grade will depend on activities performed during classes. These will include short evaluations of assigned reading, exercises performed individually or in group, and active participation. There will be two exams, one in the period 25/10/17 to 31/10/17 and another one in the period 21/12/17 to 11/01/18, each accounting for 35% of the grade.

Students that have participated in the activities associated to the ordinary evaluation but not passing the course will be offered a re-evaluation. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

Regulations for carrying out activities

Failure to perform a continuous assessment activity in the scheduled period will result in a mark of zero in that activity.
Bibliography

Basic:


Complementary: