Last update: 13-09-2017

34951 - NCA - Non-Commutative Algebra

Coordinating unit: 200 - FME - School of Mathematics and Statistics

Teaching unit: 749 - MAT - Department of Mathematics

Academic year: 2017

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)

ECTS credits: 7,5 Teaching languages: English

Teaching staff

Coordinator: ENRIC VENTURA CAPELL

Others: Primer quadrimestre:
JOSE BURILLO PUIG - A
ENRIC VENTURA CAPELL - A

Prior skills

The concept of group and subgroup, and the concept of homomorphism. Basic algebraic properties, binary operations, their properties. Equivalence relations and related set-theoretic properties.

Requirements

The basic algebra courses from the degree in mathematics.

Degree competences to which the subject contributes

Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

Teaching methodology

Classes follow the traditional structure of lecture by the professor, together with the assignment of problems and exercises for the students to solve and present, either in written or in oral form.

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Learning objectives of the subject

The main goal is to introduce the student into the basic ideas and techniques of non-commutative algebra, to the extend of being able to enroll into some initial research project in the area, if there is interest to do so.

Non-commutative algebra plays a significant role in the research panorama in mathematics today, but is underrepresented along the curriculum at the FME degree in mathematics. The main goal of the present topic is to fill this gap offering to the student a general but consistent introduction into the topic.

We'll center our attention towards the so-called "Geometric Group Theory", a relatively young and very active research area. This election is done because it allows to go, within a full semester, from the basics of the theory to the description, with a good level of details and context, of some open problems that are currently being object of active research today.

Study load

Total learning time: 187h 30m	Hours large group:	60h	32.00%
	Self study:	127h 30m	68.00%

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Content

Generalities about infinite groups

Learning time: 47h
Theory classes: 15h
Self study : 32h

Description:
The free group: basic definitions.
Presentations: generators and relations.
Short exact sequences, direct and semidirect products.
Free products, amalagams, HNN extensions.
Thompson's group as an example.

The classical Dehn problems in group theory

Learning time: 25h
Theory classes: 8h
Self study : 17h

Description:
Description of the three classical algorithmic problems in group theory: word, conjugacy and isomorphism problems.
Resolution of the word and conjugacy problems in simple cases: abelian, free, free-like constructions, residually finite, etc.
Examples of algorithmically unsolvable problems: word, membership, isomorphism problems, $F_2 \times F_2$.

The free group

Learning time: 47h
Theory classes: 15h
Self study : 32h

Description:
Stallings foldings and the lattice of subgroups of the free group.
Membership, conjugacy, finite index, intersection of subgroups.
Hall's theorem and residual properties of free groups.

Cayley graphs

Learning time: 31h
Theory classes: 10h
Self study : 21h

Description:
Cayley graph and the word metric in a group.
Dehn function, examples; characterization of the solvability of the word problem via Dehn functions.
Growth of a group, examples. Gromov theorem.

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Hyperbolic groups	Learning time: 37h 30m Theory classes: 12h Self study : 25h 30m

Description:
Definition of hyperbolic groups.
First properties, finite generation, centralizers.
Characterization of hyperbolic groups as those having linear Dehn function.

Qualification system

The student will have to develop a subject, first in term paper form, of about 15-20 pages, and also as a one to two hours lecture. The subject can be assigned by the teacher, or it can be picked by the student, among all topics in Geometric Group Theory of his interest.

Bibliography

Basic:

Brady, Noel; Riley, T.; Short, H. The geometry of the word problem for finitely generated groups. Basel: Birkhäuser, 2007. ISBN 978-3764379490.

Ghys, E.; Haefliger, A.; Verjovsky, A. Group theory from a geometrical viewpoint : 26 March - 6 April 1990, ICTP, Trieste, Italy. Singapore: World Scientific, 1991. ISBN 978-9810204426.

Lyndon, Roger C.; Schupp, Paul E. Combinatorial group theory. 2nd ed. Berlin: Springer, 2001. ISBN 978-3540411581.

Bogopolskij, Oleg Vladimirovic. Introduction to group theory. Zürich: European Mathematical Society, 2008. ISBN 9783037190418.

Complementary:

Epstein, David B.A. [et al.]. Word processing in groups. Boston: Jones and Bartlett, 1992. ISBN 978-0867202441.

Ghys, E.; La Harpe, P. de. Sur les groupes hyperboliques d'après Mikhael Gromov. Boston: Birkhäuser, 1990. ISBN 978-0817635084.