Course guide
34951 - NCA - Non-Commutative Algebra

Last modified: 29/05/2025

Unit in charge:	School of Mathematics and Statistics
Teaching unit:	749 - MAT - Department of Mathematics.

Degree:	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025   **ECTS Credits:** 7.5   **Languages:** English

LECTURER

Coordinating lecturer:	ENRIC VENTURA CAPELL
Others:	Primer quadrimestre: JOSE BURILLO PUIG - A JORGE DELGADO RODRÍGUEZ - A ENRIC VENTURA CAPELL - A

PRIOR SKILLS

The concept of group and subgroup, and the concept of homomorphism. Basic algebraic properties, binary operations and their properties. Equivalence relations and related set-theoretic properties.

REQUIREMENTS

The basic algebra courses from the degree in mathematics.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Classes follow the traditional structure of lecture by the professor, together with the assignment of problems and exercises for the students to solve and present, either in written or in oral form.

LEARNING OBJECTIVES OF THE SUBJECT

The main goal is to introduce the student into the basic ideas and techniques of non-commutative algebra, to the extend of being able to enroll himself/herself into some initial research project in the area, if there is interest to do so.

Non-commutative algebra plays a significant role in the research panorama in modern mathematics and students of any degree in mathematics have been introduced to it. The main goal of the present topic is to go a bit deeper into this area of mathematics, offering a general but consistent introduction into the topic.

We'll center our attention towards the so-called "Geometric Group Theory", a relatively young and very active research area. This election is done because it allows to go, within a full semester, from the basics of the theory to the description, with a good level of details and context, of some open problems that are currently being object of active research today.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Generalities about infinite groups

Description:
The free group: basic definitions.
Presentations: generators and relations.
Short exact sequences, direct and semidirect products.
Free products, amalagams, HNN extensions.

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

Cayley graphs and growth of groups

Description:
Cayley graph and growth of a group
Quasi-isometries, geometric properties
Growth of groups: polynomial, intermediate, exponential, uniformly exponential
Gromow theorem

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

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Subgroups of free groups

Description:
- Nielsen-Schreier theorem
- Stallings graphs
- Main properties of the lattice of subgroups of a free group
- Finite index subgroups
- Pull-back and intersection of subgroups.

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

Algorithmic problems in groups

Description:
The three classical algorithmic problems in group theory: word, conjugacy and isomorphism problems.
Resolution in simple cases: abelian, free, free-like constructions, residually finite, etc.
Tietze transformations, an attack to the isomorphism problem
Some unsolvability results: Novikov, Miller, Mihailova, etc.

Full-or-part-time: 45h
Theory classes: 15h
Self study : 30h

GRADING SYSTEM

Students will have to present in written and/or oral form some exercises assigned along the development of the course. At the end of the course, there will be a written exam with two parts: (a) the development of a theoretical topic chosen by the teachers among a closed public list of topics about the contents of the course (70%); and (b) a short list of problems/exercises about the topics of the course (30%).

BIBLIOGRAPHY

Basic:
- Rotman, Joseph. An Introduction to the theory of groups. 4th ed. New York: Springer, 1995. ISBN 0387942858.
- Lyndon, Roger C.; Schupp, Paul E. Combinatorial group theory. 2nd ed. Berlin: Springer, 2001. ISBN 9783540411581.
- Bogopolskij, Oleg Vladimirovic. Introduction to group theory. Zürich: European Mathematical Society, 2008. ISBN 9783037190418.
- Loh, Clara. Geometric group theory : an introduction [on line]. Universitext. Springer, 2017 [Consultation: 10/07/2023]. Available o n :
https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6312 482. ISBN 9783319722535.

Complementary:
- Epstein, David B.A. [et al.]. Word processing in groups. Boston: Jones and Bartlett, 1992. ISBN 9780867202441.
- Ghys, E.; La Harpe, P. de. Sur les groupes hyperboliques d'après Mikhael Gromov. Boston: Birkhäuser, 1990. ISBN 9780817635084.

RESOURCES

Other resources:
Several interesting papers and notes by Chuck Miller:

https://researchers.ms.unimelb.edu.au/ cfm/papers