34958 - MMPDE - Mathematical Modelling with Partial Differential Equations

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2017
Degree: MASTER’S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)
ECTS credits: 7,5
Teaching languages: English

Teaching staff
Coordinator: JUAN DE LA CRUZ DE SOLÀ-MORALES RUBIÓ
Others:
  Primer quadrimestre:
  XAVIER CABRE VILAGUT - A
  MATTEO COZZI - A
  JUAN DE LA CRUZ DE SOLÀ-MORALES RUBIÓ - A

Prior skills
* Good knowledge of Calculus techniques, including integral theorems and basic Complex Variable methods.
* Elementary solution of PDEs and ODEs.
* Some experience on simple cases of mathematical modelling, especially in classical physics (gravitation, heat conduction, mechanics or electromagnetism).

Degree competences to which the subject contributes

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
The course will provide a general overview on the use of partial differential equations (PDE) and boundary value problems to construct mathematical models of real phenomena.

By the end of the course the student should have acquired:

* a knowledge of the problems that can be modelled with PDE's.
* intuitive and physical interpretations of the terms that appear on PDE's.

### Study load

<table>
<thead>
<tr>
<th>Total learning time: 187h 30m</th>
<th>Hours large group:</th>
<th>60h</th>
<th>32.00%</th>
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<tbody>
<tr>
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<td>Self study:</td>
<td>127h 30m</td>
<td>68.00%</td>
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**Teaching methodology**

Lectures will contain the main contents of the course, but the students will also be asked to make presentations of additional material in seminar sessions. Problem solution will also be asked.
## Content

<table>
<thead>
<tr>
<th>Section</th>
<th>Learning time: 56h 15m</th>
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| **1 Heat conduction and diffusion** | Theory classes: 18h  
Self study: 38h 15m |
| **2 Potentials in physics and technology** | Theory classes: 18h  
Self study: 38h 15m |
| **3 Transients in continuous media** | Theory classes: 10h  
Self study: 21h 15m |
| **4 Geometry** | Theory classes: 7h 30m  
Self study: 15h 56m |

### Description:
- **1 Heat conduction and diffusion**
  - Boundary conditions, Energy Functionals, separation of variables, Thin domains, Convergence to gaussians, entropy
  - Steffan Problem, Black-Scholes model, Reaction-diffusion

- **2 Potentials in physics and technology**
  - Classical gravitation, electrostatics, volume and layer potentials
  - Euler equations of inviscid fluids and potential flows.
  - Complex analysis methods in plane potential flows. Lift and drag.
  - Navier-Stokes system and the viscous contribution to drag. Stokes and Boundary layer equations.

- **3 Transients in continuous media**
  - Acoustics, surface gravity waves, inertial waves.
  - Electromagnetic and elastic waves.
  - Dispersion, Stationary waves and high-frequency waves.

- **4 Geometry**
  - The Laplace-Beltrami operator.
  - Minimal surfaces.
5 Calculus of Variations

<table>
<thead>
<tr>
<th>Learning time: 23h 26m</th>
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<tbody>
<tr>
<td>Theory classes: 7h 30m</td>
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<tr>
<td>Self study: 15h 56m</td>
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Description:
Calculus of Variations and Euler-Lagrange Equations
Other minimization problems

Qualification system
Attendance to lectures, presentation of additional materials and problem solving will be the basis of a qualification up to a certain level (60%). A higher mark will require a written exam.

Bibliography

Basic:

Complementary: