Last update: 15-06-2018

34962 - HS - Hamiltonian Systems

Coordinating unit: 200 - FME - School of Mathematics and Statistics

Teaching unit: 749 - MAT - Department of Mathematics

Academic year: 2018

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)

ECTS credits: 7,5 Teaching languages: English

Teaching staff

Coordinator: MARCEL GUARDIA MUNARRIZ

Others: Segon quadrimestre:
AMADEU DELSHAMS I VALDES - A
MARCEL GUARDIA MUNARRIZ - A

Prior skills

Knowledge of calculus, algebra and ordinary differential equations.

Degree competences to which the subject contributes

Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATI0N RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

Teaching methodology

Standard exposition in front of the blackboard, resolution of exercices, completion of a project and attendance to the JISD summer school http://www.ma1.upc.edu/recerca/jisd

Learning objectives of the subject

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To comprehend the basic foundations of the theory of Hamiltonian systems, and to understand its applications to Celestial Mechanics and other fields.

Study load

Total learning time: 187h 30m	Hours large group:	60h	32.00%
	Self study:	127h 30m	68.00%

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Content

Hamiltonian formalism	Learning time: 28h Theory classes: 10h Self study : 18h

Description:
Hamiltonian dynamical systems: symplectic maps, symplectic manifolds. Linear Hamiltonian systems and their application to the study of stability of equilibrium points. Canonical transformations.

Celestial mechanics	Learning time: 34h Theory classes: 12h Self study : 22h

Description:
The two body problem, first integrals. Resolution. The three body problem, different coordinates. The restricted three body problem. Central configurations. Periodic orbits, invariant manifolds.

Geometric theory and invariant objects of Hamiltonian systems	Learning time: 24h Theory classes: 8h Self study : 16h

Description:
Continuous and discrete dynamical systems, Poincaré map. Flow box Theorem. Noether Theorem. Periodic orbits. Continuation of periodic orbits. Lyapunov Center Theorem.

Integrable systems	Learning time: 10h Theory classes: 4h Self study : 6h

Description:
Complete integrability and Liouville-Arnold theorem. Action-Angle coordinates. Quasi-periodic flows on a torus, resonances.

Quasi-integrable Hamiltonian systems	Learning time: 26h Theory classes: 8h Self study : 18h

Description:
Examples of quasi-integrable systems. Small divisors and Diophantine inequalities. Averaging Theory. Lie Method. KAM Theory (Kolmogorov-Arnold Moser). Effective stability and Nekhoroshev theorem. Melnikov Potential. Arnold diffusion.

Lagrangian systems and variational methods	Learning time: 12h Theory classes: 4h Self study : 8h

Description:
Lagrangian systems. Legendre transformation. Principle of minimal action. Twist maps. Existence of periodic orbits. Aubry-Mather Theory.

Hamiltonian Partial Differential Equations	Learning time: 4h Theory classes: 2h Self study : 2h

Description:
Linear Hamiltonian Partial Differential Equations. Examples. Periodic, quasi-periodic and almost-periodic solutions. Nonlinear Hamiltonian Partial Differential Equations. Lyapunov stability/instability of invariant objects. Transfer of energy.

- Interactions between Dynamical Systems and Partial Differential Equations	Learning time: 49h 30m Theory classes: 12h Self study : 37h 30m

Description:
Summer School and Research workshop on topics between Dynamical Systems and Partial Differential Equations

Planning of activities

JISD summer school

Description:
Attendance to the JISD summer school

Specific objectives:
To learn from oustanding researchers a view of the state of the art in several research topics, interacting with students of the rest of Spain and of the World.

Qualification system

The students have to do some problems and a project. There will be also an exam of the theoretical part of the course. Moreover, they will attend the JISD.

Bibliography

Basic:

Meyer, Kenneth R.; Hall, Glen R.; Offin, Dan. Introduction to Hamiltonian dynamical systems and the n-body problem. 2nd ed. New York: Springer-Verlag, 2009. ISBN 978-0-387-09723-7.

Arnol'd, V. I.; Kozlov, Valerii V.; Neishtadt, Anatoly I. Mathematical aspects of classical and celestial mechanics [on line]. 3rd ed. Berlin: Springer-Verlag, 2006Available on: <http://dx.doi.org/10.1007/978-3-540-48926-9>. ISBN 3540282467.

Treschev, Dmitry; Zubelevich, Oleg. Introduction to the perturbation theory of Hamiltonian systems. Berlin: Springer Verlag, 2010. ISBN 978-3-642-03027-7.

Celletti, Alessandra. Stability and chaos in celestial mechanics [on line]. Springer-Praxis, 2010Available on: <http://site.ebrary.com/lib/upcatalunya/detail.action?docID=10372372>. ISBN 978-3-540-85145-5.

Wintner, Aurel. The analytical foundations of celestial mechanics. Dover Publications, ISBN 978-0486780603.

Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1997. ISBN 9780521575577.

Berti, Massimiliano. Nonlinear Oscillations of Hamiltonian PDEs [on line]. Boston, MA: Birkhäuser Boston, Inc., 2007Available on: <http://dx.doi.org/10.1007/978-0-8176-4681-3>. ISBN 978-0-8176-4680-6.

Marsden, Jerrold E; Ratiu, Tudor S. Introduction to mechanics and symmetry : a basic exposition of classical mechanical systems. 2a ed. New York [etc.]: Springer, 1999. ISBN 978-0-387-98643-2.

Kanuf, Andreas. Mathematical physics: Classical mechanics. 1. Springer-Verlag, 2018. ISBN 978-3-662-55772-3.

Others resources:

Hyperlink

Grup de sistemes dinàmicshttps://recerca.upc.edu/sd

Pàgina web del Grup de Sistemes Dinàmics de la UPC on es descriuen diversos projectes i els investigadors que hi treballen així com diverses activitats relacionades