34966 - VD - Differentiable Manifolds

**Coordinating unit:** 200 - FME - School of Mathematics and Statistics

**Teaching unit:** 749 - MAT - Department of Mathematics

**Academic year:** 2017

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)

**ECTS credits:** 7,5  
**Teaching languages:** English

### Teaching staff

**Coordinator:** EVA MIRANDA GALCERÁN

**Others:** Segon quadrimestre:  
EVA MIRANDA GALCERÁN - A  
MIGUEL CARLOS MUÑOZ LECANDA - A

### Prior skills

Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with a lesser knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

### Degree competences to which the subject contributes

#### Specific:

1. **RESEARCH.** Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.

2. **CALCULUS.** Obtain (exact or approximate) solutions for these models with the available resources, including computational means.

3. **CRITICAL ASSESSMENT.** Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

4. **SELF-DIRECTED LEARNING.** Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

5. **EFFICIENT ORAL AND WRITTEN COMMUNICATION.** Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.

6. **THIRD LANGUAGE.** Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.

7. **EFFECTIVE USE OF INFORMATION RESOURCES.** Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
34966 - VD - Differentiable Manifolds

Teaching methodology

Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.

Learning objectives of the subject

The subject focuses on some of the fundamental topics of differential geometry and its applications in different areas, as geometric mechanics, control theory, classic and quantum field theory, fluid mechanics, computer vision, geophysical dynamics, general relativity and more.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 187h 30m</th>
<th>Hours large group:</th>
<th>60h</th>
<th>32.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self study:</td>
<td>127h 30m</td>
<td>68.00%</td>
<td></td>
</tr>
</tbody>
</table>

Last update: 02-11-2017
### Content

<table>
<thead>
<tr>
<th>Topic</th>
<th>Learning time:</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reminder of Manifold Theory and Exterior</td>
<td>12h 52m</td>
<td>Brief survey of manifold theory and differential geometry. Manifolds, atlases, smooth maps, tangent vectors and vector fields, flows, exterior calculus.</td>
</tr>
<tr>
<td>Calculus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lie groups and Lie algebras. Actions on</td>
<td>25h</td>
<td>Introduction to the main aspects of the theory of Lie groups and their actions on manifolds, including classic groups, subgroups, actions, orbits and quotients.</td>
</tr>
<tr>
<td>Manifolds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Principal Bundles</td>
<td>18h 45m</td>
<td>The concept of fibre bundles and local triviality will be introduced. Then we define the main object of study, principal bundles and their main example, frame bundles, as well as their properties.</td>
</tr>
<tr>
<td>Connections and Curvature</td>
<td>18h 45m</td>
<td>We introduce connections on principal bundles and study their existence and main constructions and properties, as curvature, holonomy, parallelism and structure equations.</td>
</tr>
</tbody>
</table>

**Description:**

- **Reminder of Manifold Theory and Exterior Calculus:**
  - Theory classes: 4h
  - Self study: 8h 52m

- **Lie groups and Lie algebras. Actions on Manifolds:**
  - Theory classes: 8h
  - Self study: 17h

- **Principal Bundles:**
  - Theory classes: 6h
  - Self study: 12h 45m

- **Connections and Curvature:**
  - Theory classes: 6h
  - Self study: 12h 45m
There will be a final exam, as well as the possibility to write an optional essay that would contribute to the final grade. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests.

### Qualification system

We will study constructions in bundle theory, as associated and pullback bundles, and the theory of general vector bundles. The main objective is to introduce connections on vector bundles and their properties, as well as their relationship with connections on principal bundles. Tubular neighbourhood theorem. Introduction to Differential Topology.

**Description:**
We will define De Rham cohomology and compare to other cohomologies. We will also introduce De Rham computation kit and Poincaré duality.

**Description:**
Introduction to symplectic and Poisson manifolds with emphasis on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem or the Lagrangian neighbourhood theorem. Special attention will be given to examples provided by the realm of integrable systems. We end the chapter introducing the basic concepts in Poisson geometry.

---

**Vector Bundles and Associated Bundles**

**Learning time:** 18h 45m
- Theory classes: 6h
- Self study: 12h 45m

**Description:**
We will study constructions in bundle theory, as associated and pullback bundles, and the theory of general vector bundles. The main objective is to introduce connections on vector bundles and their properties, as well as their relationship with connections on principal bundles. Tubular neighbourhood theorem. Introduction to Differential Topology.

**De Rham Cohomology and Integration Theory**

**Learning time:** 25h
- Theory classes: 8h
- Self study: 17h

**Description:**
We define De Rham cohomology and compare to other cohomologies. We will also introduce De Rham computation kit and Poincaré duality.

**Symplectic and Poisson Geometry**

**Learning time:** 43h 45m
- Theory classes: 14h
- Self study: 29h 45m

**Description:**
Introduction to symplectic and Poisson manifolds with emphasis on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem or the Lagrangian neighbourhood theorem. Special attention will be given to examples provided by the realm of integrable systems. We end the chapter introducing the basic concepts in Poisson geometry.
Regulations for carrying out activities

The final grade awarded to the student would we computed as follows:

- Case A: an student that does only the final exam. Then the final grade would be that of the exam.
- Case B: an student that does the final exam AND submits a written essay. Then the final note would be the result of $\text{MAX}(\text{exam}, 40\% \text{ exam} + 60\% \text{ (essay+ exercises)})$

Bibliography

Basic:

Guillemin, Victor; Pollack, Alan. Differential topology. Reprint of the 1974 original. AMS Chelsea Publishing,

Complementary: