

Course guide

200203 - VD - Differentiable Manifolds

Last modified: 11/04/2024

Unit in charge:	School of Mathematics and Statistics		
Teaching unit:	749 - MAT - Department of Mathematics.		
Degree:	BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).		
Academic year: 2024	ECTS Credits: 6.0	Languages: Catalan	

LECTURER

Coordinating lecturer:	FRANCESC XAVIER GRACIA SABATE
Others:	Primer quadrimestre: CARLES BATLLE ARNAU - M-A FRANCESC XAVIER GRACIA SABATE - M-A

PRIOR SKILLS

All the capacities included in the subjects of Linear Algebra, Multilinear Algebra, Calculus, Differential Calculus, Integral Calculus, Topology, Differential Geometry and Ordinary Differential Equations.

REQUIREMENTS

Having passed the subjects listed in the section on previous skills.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:

3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generical:

1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:

11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

The course content will be presented and developed in the lectures. Most topics will be presented by the instructors, but there may be some sessions devoted to student presentations.

A list of problems will be designed to help students to deepen and extend their command of concepts and techniques presented in the lectures. Some problems will be solved in class and others by the students, who will deliver their solutions.

LEARNING OBJECTIVES OF THE SUBJECT

The main objectives of the course are:

- To understand and master the basic concepts of differential geometry: differentiable manifolds, differentiable mappings, tangent and cotangent spaces, tangent mapping, submanifolds, vector fields and differential 1-forms, tensor fields, etc..
- To perform basic calculations with the objects mentioned in both coordinate form and intrinsically.
- To understand the geometric interpretation of the objects studied and relate them to previously studied subjects, such as differential calculus, integral calculus, linear and multilinear algebra, differential geometry of curves and surfaces and differential equations, as well as those implemented in parallel such as topology or algebraic geometry.

Furthermore, at the end of the course, students should:

- Be able to find appropriate literature and understand the scientific literature on the subject.
- Be able to apply the concepts studied in other areas, such as theoretical mechanics, control theory, mathematical physics or geometry of dynamical systems.
- Be aware of the wide range of fields and problems where the techniques of differential geometry can be applied.
- Be able to join a research group on these topics and their applications.

STUDY LOAD

Type	Hours	Percentage
Hours large group	30,0	20.00
Self study	90,0	60.00
Hours small group	30,0	20.00

Total learning time: 150 h

CONTENTS

Basic topics.

Description:

- 1 - Differentiable manifolds. Tangent bundle. Vector fields and flows. Lie derivative. Submanifolds and differentiable mappings.
- 2 - Introduction to Lie groups and Lie algebras. Classical Lie groups and Lie algebras.
- 3 - Tangent distributions and foliations. Frobenius theorem and applications.
- 4 - Riemannian geometry. Levi-Civita connection. Covariant derivatives. Geodesics and exponential mapping. Curvature. Hopf-Rinow theorem.
- 5 - Cotangent bundle. Differential forms. Tensor fields. Introduction to the de Rham cohomology. Pfaff systems.

Full-or-part-time: 60h

Theory classes: 30h

Practical classes: 30h

GRADING SYSTEM

The evaluation of the subject stems from deliverables (20%), a brief control (10%) and a final exam with theory questions (20%) and problems (50%). If the marks of the control are lower than those of the final exam of problems, then the respective weights will be 0%, 60%.

In the case of a small group, the written exam can be replaced by personal work and oral presentations.

BIBLIOGRAPHY

Basic:

- Conlon, L. Differentiable manifolds. 2nd ed. Boston: Birkhauser, 2008. ISBN 9780817647667.
- Carmo, Manfredo Perdigão do. Riemannian geometry. Boston: Birkhäuser, 1992. ISBN 0817634908.
- Lee, John M. Introduction to smooth manifolds. 2nd. New York: Springer, cop. 2013.
- Lee, John M. Riemannian manifolds: an introduction to curvature [on line]. New York: Springer, 1997 [Consultation: 21/06/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b98852>. ISBN 038798271X.
- Tu, Loring W. An introduction to manifolds [on line]. 2nd ed. New York: Universitext, Springer, 2010 [Consultation: 21/06/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-7400-6>. ISBN 9780387480985.

Complementary:

- Abraham, R.; Marsden, J.; Ratiu, T. Manifolds, tensor analysis and applications. 2nd ed. New York: Springer-Verlag, 1988. ISBN 0387967907.
- Aubin, Thierry. A course in differential geometry. Providence: American Mathematical Society, 2001. ISBN 082182709X.
- Boothby, William Munger. An introduction to differentiable manifolds and riemannian geometry. 2nd ed. San Diego: Academic Press, 1986. ISBN 012116053X.
- Wasserman, Robert H. Tensors and manifolds with applications to physics. 2nd ed. Oxford: Oxford University Press, 2004. ISBN 0198510594.
- Lafontaine, Jacques. Introduction aux variétés différentielles. Presses Universitaires de Grenoble, 1996. ISBN 2706106549.
- Warner, Frank W. Foundations of differentiable manifolds and lie groups. Springer, 1983.
- Gadea, Pedro M.; Muñoz Masqué, Jaime; Mykytyuk, Ihor V. Analysis and algebra on differentiable manifolds [on line]. 2nd. London: Springer, 2013 [Consultation: 21/06/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-94-007-5952-7>. ISBN 9789400759510.

RESOURCES

Hyperlink:

- Pàgina amb informació i materials del curs. <https://web.mat.upc.edu/xavier.gracia/vardif/>