

## Course guides

### 200243 - LF - Logic and Foundations

Last modified: 17/05/2013

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 726 - MA II - Department of Applied Mathematics II.

**Degree:** BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

**Academic year:** 2013    **ECTS Credits:** 6.0    **Languages:** Catalan

#### LECTURER

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**Coordinating lecturer:** RAIMON ELGUETA MONTO

**Others:**

RAIMON ELGUETA MONTO - A  
FRANCESC TIÑENA SALVAÑA - A

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

**Generical:**

1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

**Transversal:**

11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

#### TEACHING METHODOLOGY

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(Section not available)

#### LEARNING OBJECTIVES OF THE SUBJECT

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(Section not available)



## STUDY LOAD

Type	Hours	Percentage
Hours large group	30,0	20.00
Hours small group	30,0	20.00
Self study	90,0	60.00

Total learning time: 150 h

## CONTENTS

### 1. Introduction: The Hilbert program for the foundations of mathematics

**Full-or-part-time:** 1h

Theory classes: 1h

### 2. Background: The transformation of mathematics in the 19th century

**Description:**

A. Mathematical objects.

Preliminaries: Numbers and magnitudes in Greek mathematics. The construction of the reals. Transformation of the notion of integer. The extension of finite numbers: transfinite numbers and set theory. The ontological and methodological debate: paradoxes of the infinite.

B. The evolution of the method.

Introduction: certainty of mathematical knowledge, the use of the axioms and language. The axiomatic method: from Greek deductive mathematics to Hilbert's conception. The emergence of symbolic logic: from sillogistics to the symbolic calculi of the end of 19th century.

**Full-or-part-time:** 21h

Theory classes: 11h

Guided activities: 3h

Self study : 7h

### 3. Hilbert's program: context and development

**Description:**

A. Formulation: 1900-1921

Situation in the turn of the century: the crises of foundations. Period 1900-1905: Hilbert's lecture in Paris, the first outline of the program in 1905 and Poincaré's critiques. The alternatives: Russell's logicist solution, axiomatic proposal by Zermelo and Brouwer and Weyl's intuitionism. Hilbert's reaction: The emergence of first order logic and the formulation of the program in 1921.

B. Contributions: 1921-1936

The completeness of logic: Gödel's theorem, compactness and Skolem's paradox. The decision problem: Notion of algorithm and the undecidability of first order logic. The incompleteness phenomenon and the necessity of a reformulation of Hilbert's program. Proofs of consistency: The extension of finitary methods and Gentzen's consistency proof for arithmetic.

**Full-or-part-time:** 70h

Theory classes: 25h

Guided activities: 10h

Self study : 35h

## GRADING SYSTEM

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The course grade (N) is obtained from:

- \* Delivery of exercises during the course (P) (they consist in a written brief discussion of a theme proposed by the teacher or in solving a problem), and
- \* A final exam (F).

Then,  $N = 0.4P + 0.6 F$ .

## BIBLIOGRAPHY

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### Basic:

- Enderton, Herbert B. A mathematical introduction to logic. 2nd ed. San Diego (Calif.) [etc.]: Harcourt / Academic Press, 2002. ISBN 0122384520.
- Fraenkel, Abraham Adolf [ et al.]. Foundations of set theory. 2nd. revised edition. Amsterdam [etc.]: North-Holland, 1984. ISBN 0720422701.
- Katz, Victor J. A History of mathematics : an introduction. 2nd ed., with corrections. Reading,[etc.]: Addison-Wesley, 1998. ISBN 0321016181.
- Kneebone, G.T. Mathematical logic and the foundations of mathematics : an introductory survey. Mineola: Dover Publications, 2001. ISBN 048617123.

### Complementary:

- Murawski, Roman. Recursive functions and metamathematics : problems of completeness and decidability, Godel's theorems. Dordrecht: Kluwer Academic Publishers, cop. 1999. ISBN 0792359046.
- Ewald, William [ed]. From Kant to Hilbert: a source book in the foundations of mathematics. Oxford: Clarendon Press, 1996. ISBN 0198532717.
- Feferman, Solomon. In the light of logic. New York [etc.]: Oxford University Press, 1998. ISBN 0195080300.
- Ferreirós Domínguez, José. Labyrinth of thought : a history of set theory and its role in modern mathematics. Basel: Birkhäuser Verlag, 1999. ISBN 3764357495.
- Guiaquinto, M. The Search for certainty : a philosophical account of foundations of mathematics. Oxford: Clarendon Press, 2002. ISBN 019875244X.
- Hilbert, D. ; Bernays, P. Fondements des mathématiques. Paris: L'Harmattan, 2001. ISBN 2747515184.
- Stillwell, John. Roads to infinity : the mathematics of truth and proof. Natick, Mass: AK Peters, cop. 2010. ISBN 9781568814667.
- Van Heijenoort, Jean. From Frege to Gödel : a source book in mathematical logic : 1979-1931. Cambridge, Mass. [etc.]: Harvard Univ. Press, 1981.
- Gray, Jeremy. Plato's ghost [Recurs electrònic] : the modernist transformation of mathematics [on line]. Princeton, N.J.: Princeton University Press, 2008 [Consultation: 30/05/2012]. Available on: <http://site.ebrary.com/lib/upcatalunya/docDetail.action?docID=10359260>. ISBN 9780691136103.