Course guides
250101 - ALGGEOM - Algebra and Geometry

Last modified: 01/10/2021

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2010). (Compulsory subject).
BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2017). (Compulsory subject).

Academic year: 2021 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: MARIA ANGELES PUIGVI BURNIOL

Others: MARIA ANGELES PUIGVI BURNIOL

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
3048. Ability to solve the types of mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

Transversal:
591. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
597. EFFECTIVE USE OF INFORMATI0N RESOURCES - Level 1. Identifying information needs. Using collections, premises and services that are available for designing and executing simple searches that are suited to the topic.
600. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

TEACHING METHODOLOGY

The course consists of 4 hours a week of classes in the classroom.

We spend 2.5 hours in lectures, in which the teacher explains the concepts and basic materials of the subject, presents examples and exercises.

1.5 hours is devoted to solving problems with greater interaction with students. Exercises are conducted to consolidate the general and specific learning objectives.

Support materials used in the form of detailed teaching plan by algweb.net/2.0/ page: content, programming and evaluation activities directed learning and literature.

LEARNING OBJECTIVES OF THE SUBJECT

Students will acquire a general understanding of linear algebra, analytical geometry in two and three dimensions, and methods for solving linear problems encountered in engineering. They will also develop the skills to analyse and solve mathematical problems in engineering that involve these concepts.

On completion of the course, students will have acquired the ability to:
1. Interpret vector spaces;
2. Solve linear equation systems manually and using basic software;
3. Produce geometric interpretations of concepts in vector calculus;
4. Use algebraic methods applicable to vectors, matrices, operators and tensors, including basic operations, reduction to canonical form and change of base.

Logic, set theory and algebraic structures; Vector spaces, including matrix algebra; Systems of linear equations, linear applications and bilinear forms and the basic algorithms used to solve them; Euclidean spaces; Determinants and their applications, in particular for calculating areas and volumes; Analytical geometry; Linear operators: Endomorphisms and spectral theorems, affine Euclidean spaces, eigenvalues and eigenvectors; Tensor algebra: Basic operations, change of base and tensor calculus

Delve into the mechanisms of logical reasoning. Studying methods of solving linear problems that appear frequently in engineering. Submit items tensor algebra and analytic geometry.

STUDY LOAD

Type	Hours	Percentage
Hours large group	30,0	20.00
Guided activities	6,0	4.00
Hours medium group	15,0	10.00
Self study	84,0	56.00
Hours small group	15,0	10.00

Total learning time: 150 h

CONTENTS

Unit 1: Linear maps

Description:
Definitions and examples. Image and Kernel subspaces. Monomorphism, epimorphism, isomorphism. Basic properties. Vector space of linear maps. Maps over and onto finite dimensional vector spaces. Associated matrix. Computation of basis for the image and the kernel of a linear map.
Fundamental theorem.
The aim is to solve some problems on linear maps defined over infinite dimensional vector spaces, although focus shall be primarily on the finite dimensional case. Using the associated matrix and the theory of vector spaces and linear systems, basis for the kernel and image shall be obtained.
Composition of linear maps. Ring of endomorphisms. Inverse of an isomorphism. Matrix associated with a composition. Dual vector space. Basis change in a vector space. Connection of base change with the matrix associated with a linear map.
Problems on the composition of linear maps. Computation of the matrix associated with the inverse map of an isomorphism. Base of the dual vector space.

Specific objectives:
Introduce the fundamental concepts and relate them to the mathematical contents covered in other courses. Acquaint the student with vectors other than the familiar examples deriving from physical applications. Relate the injectivity and surjectivity of a linear map to its kernel and image.
Relate new concepts to the student's general background: vector spaces, matrix properties, resolution of linear systems, rang of a vector system and implicit equations of a subspace.
Justify the correspondence between invertible and bijective linear maps. Relate the composition of linear maps to the product of their associated matrices. Introduce the concept of dual space. Computation of basis and analysis of their properties. Highlight the theory of base change, which shall be paramount in subsequent units: pay special attention to the relation between vector components in different basis and base vectors.
Relate the concepts covered in this unit to familiar matrix properties. Focus on the dual vector space and on the computation of basis. Draw the student's attention to geometric applications.

Full-or-part-time: 14h 23m
Theory classes: 4h
Practical classes: 2h
Self study : 8h 23m

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Unit 2: Euclidean space

Description:
Bilinear forms. Examples and basic properties. Matrix associated to a bilinear form. Changing the base. Symmetric bilinear form. Quadratic form. Definite forms. Canonical form and normal form of a real symmetric bilinear form.
Problems are solved by reduction of a symmetric bilinear form to its canonical form and normal. Change of basis.
Definition of real and complex inner product. Examples. Basic properties. Orthogonal subspace. Orthogonal and orthonormal basis. Orthogonal projection. Pythagoras theorem and law of the parallelepiped.
Fourier coefficient. Schwarz inequality, Bessel and triangular. Method of Gram-Schmidt. Geometric interpretations.
Properties of real and complex inner product. Orthogonal projection onto a subspace. Geometric interpretations.

Specific objectives:
To develop the properties of bilinear forms, especially the symmetrical, preparing its subsequent application in Calculus.
Working properties of symmetric bilinear forms using the method of elementary row operations they already know. Properties of operations with complex numbers.
Present the definitions and general properties and continually interpret in real Euclidean space of three dimensions with which the student is familiar. It aims at an abstract knowledge of Euclidean space applications are working especially geometry.
Present and demonstrate the fundamental properties. Interpreting real Euclidean space of three dimensions. Justify geometric properties.
Acquiring skill in the demonstration and use of abstract properties. Operate with complex numbers, general properties apply to real three-dimensional Euclidean space.

Full-or-part-time: 28h 47m
Theory classes: 6h
Practical classes: 2h
Laboratory classes: 4h
Self study : 16h 47m

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Unit 3: Determinants

Description:
Definition of a determinant. Fundamental properties. Determinant of a triangular matrix. Determinant of a block diagonal matrix. Calculation of determinants. Gauss method.
Examples of determinants calculated by reducing the matrix to triangular form.
Expression of the determinant. Development of a row and a column. Determinant of matrix multiplication. Determinants and matrix inversion. Cramer's rule.
Exercises to determine if a matrix is invertible, and if so get its inverse. Solving systems of linear equations using Cramer's Rule.
Geometric applications: base change matrix from orthonormal bases. Volume of a parallelepiped. Vector product. Properties.

Specific objectives:
Define the alternating multilinear forms, of which the determinant is a special case. From its definition shows some basic properties, all without having to explain the development of the determinant. Calculate the determinant of a matrix by applying numerical row elementary operations to reduce it to the triangular shape.
Practicing proper elementary row operations and become aware of the possible numerical programming method.
After introducing the basic properties of permutations make explicit the crucial and its development. Using the properties of alternating multilinear forms of showing that the determinant of a matrix is the product of determinants. Cofactor matrix is defined and used in the calculation of the inverse matrix.
Working this second method to invert a matrix, as already know to transform it into reduced row echelon form by row.
Provide the student with the tools they need in calculus, and to justify the existence of polynomial interpolators.

Full-or-part-time: 14h 23m
Theory classes: 4h
Practical classes: 2h
Self study : 8h 23m

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Full-or-part-time: 14h 23m
Theory classes: 4h
Practical classes: 2h
Self study : 8h 23m

Unit 4: Endomorphism and matrix reduction

Description:
Eigenvalues and eigenvectors. Characteristic polynomial. Diagonalization general theorem.
Diagonalization elementary theorem. Examples.
Diagonalization exercises.
Trigonalization basic theorem. Examples. Cayley-Hamilton. Examples and applications.
Trigonalization problems.
Exercises. Part 3.

Full-or-part-time: 21h 36m
Theory classes: 6h
Practical classes: 3h
Self study : 12h 36m

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Eigenvalues and eigenvectors. Characteristic polynomial. Diagonalization general theorem.
Diagonalization elementary theorem. Examples.
Diagonalization exercises.
Trigonalization basic theorem. Examples. Cayley-Hamilton. Examples and applications.
Trigonalization problems.
Exercises. Part 3.

Full-or-part-time: 21h 36m
Theory classes: 6h
Practical classes: 3h
Self study : 12h 36m

Unit 5: Operators and spectral theorems.

Description:
Definitions and basic properties. Transposed. Relationship with bilinear forms. Associated matrices. Normal operators. Properties. Spectral theorem for normal operators.
Some types of normal operators, characterization and properties. Hermitian operators, skew-hermitian and Unitary. Real normal operators. Symmetric operators, skew-symmetric and orthogonal. Spectral theorem for symmetric operators.
Problems of normal operators and properties of normal matrices.
Canonical form of real nonsymmetric normal operators. Isometries. Classification of orthogonal operators in three dimensional space.
Problems of real normal operators.

Full-or-part-time: 33h 36m
Theory classes: 5h
Practical classes: 4h
Laboratory classes: 5h
Self study : 19h 36m

Unit 5: Operators and spectral theorems.

Description:
Definitions and basic properties. Transposed. Relationship with bilinear forms. Associated matrices. Normal operators. Properties. Spectral theorem for normal operators.
Some types of normal operators, characterization and properties. Hermitian operators, skew-hermitian and Unitary. Real normal operators. Symmetric operators, skew-symmetric and orthogonal. Spectral theorem for symmetric operators.
Problems of normal operators and properties of normal matrices.
Canonical form of real nonsymmetric normal operators. Isometries. Classification of orthogonal operators in three dimensional space.
Problems of real normal operators.

Full-or-part-time: 33h 36m
Theory classes: 5h
Practical classes: 4h
Laboratory classes: 5h
Self study : 19h 36m

Unit 6: Tensor algebra.

Description:
Dual space. Tensors related: multilinear forms. Definitions and basic properties. Basis change. Notations. Tensor product. Detailed study of tensors of order one and two.
Higher-order tensors. Contraction symmetric tensors, skew-symmetric and completely skew-tisymmetric. Exterior product. Cartesian tensors. Tensor of inertia.
Tensors problems and their physical applications.

Full-or-part-time: 26h 24m
Theory classes: 5h
Practical classes: 2h
Laboratory classes: 4h
Self study : 15h 24m

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Higher-order tensors. Contraction symmetric tensors, skew-symmetric and completely skew-tisymmetric. Exterior product. Cartesian tensors. Tensor of inertia.
Tensors problems and their physical applications.

Full-or-part-time: 26h 24m
Theory classes: 5h
Practical classes: 2h
Laboratory classes: 4h
Self study : 15h 24m

Tests

Full-or-part-time: 4h 48m
Laboratory classes: 2h
Self study : 2h 48m

Tests

Full-or-part-time: 4h 48m
Laboratory classes: 2h
Self study : 2h 48m

GRADING SYSTEM

The final grade is obtained from partial qualifications follows:

E0: continuous assessment activities
E1: Test of the units developed on the first half of the academic term
E2: Test of the units developed on the second half of the academic term
E3: Global test of the course

The student has to choose whether to take test E2 or E3

NF1=0.3E0+0.35E1+0.35E2

NF2=0.3E0+0.7E3

Final Mark = max {NF1, NF2}

The exams consist of a part with questions on concepts associated with learning objectives in terms of subject knowledge or understanding, application and a set of exercises

Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation tests will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

These tests must be authorized by the corresponding Head of Studies, at the request of the professor responsible for the course, and will be carried out within the corresponding academic period.

EXAMINATION RULES.

Failure to perform a laboratory or continuous assessment activity in the scheduled period will result in a mark of zero in that activity.

BIBLIOGRAPHY

Basic:

- Rojo, J. Álgebra lineal. 2a ed. Madrid: McGraw-Hill, 2007. ISBN 9788448156350.
- Hoffman, K.; Kunze, R. Álgebra lineal. México D.F. [etc.]: Prentice Hall Hispanoamericana, 1973. ISBN 9688800090.
- Proskuriakov, I. Problemas de algebra Lineal. Moscow: Mir, 1986.
- Rojo, J.; Martín, I. Ejercicios y problemas de álgebra lineal. 2a ed. Madrid: McGraw-Hill, 2005. ISBN 8448198581.