

## Course guide

# 250102 - CÀLCUL - Calculus

Last modified: 01/10/2021

**Unit in charge:** Barcelona School of Civil Engineering  
**Teaching unit:** 751 - DECA - Department of Civil and Environmental Engineering.

**Degree:** **Academic year:** 2021 **ECTS Credits:** 9.0  
**Languages:** Catalan, Spanish

### LECTURER

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**Coordinating lecturer:** M. ROSA ESTELA CARBONELL

**Others:** M. ROSA ESTELA CARBONELL, FRANCISCO JAVIER MARCOTE ORDAX

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

3048. Ability to solve the types of mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

3049. Ability to select resources from knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods; numerical algorithms; statistics and optimisation. All with a view to solving the types of mathematical problems that may arise in engineering

#### Transversal:

588. SUSTAINABILITY AND SOCIAL COMMITMENT - Level 1. Analyzing the world's situation critically and systemically, while taking an interdisciplinary approach to sustainability and adhering to the principles of sustainable human development. Recognizing the social and environmental implications of a particular professional activity.

591. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.

598. EFFECTIVE USE OF INFORMATION RESOURCES - Level 2. Designing and executing a good strategy for advanced searches using specialized information resources, once the various parts of an academic document have been identified and bibliographical references provided. Choosing suitable information based on its relevance and quality.

601. SELF-DIRECTED LEARNING - Level 2: Completing set tasks based on the guidelines set by lecturers. Devoting the time needed to complete each task, including personal contributions and expanding on the recommended information sources.

### TEACHING METHODOLOGY

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The course consists of 7 hours per week of classroom activity from beginning the of November to the end of the first four-month period. During the second quarter there are 3 hours a week of classroom.

In the course there are theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises and others with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

The course uses the "flipped classroom" methodology where the student, by means of specific group-dynamics techniques, extends and consolidates the knowledge acquired during the out-of-class preparation, in advance, of basic elements corresponding the following classes. The out-of-class preparation is carried out by the student, supported by videos, transparencies, books and bibliographic material, provided on the website of the course, and according to the directions of the teacher. Then, the in-class group dynamics consists of providing the group of students the required additional knowledge, according to the possible weaknesses identified by the teacher, perform practical exercises, answer questions, deepen the students knowledge on the subject and promote teamwork.

Support material is provided through the virtual campus ATENEA in the form of detailed activity plans which include contents, learning programme, bibliography and assessment activities.



## LEARNING OBJECTIVES OF THE SUBJECT

Students will learn to perform differential and integral calculus of one variable and to apply these techniques to mathematical problems encountered in engineering.

On completion of the course, students will have acquired the ability to:

1. Relate ordinary differential equations to engineering problems, solve ODEs in simple conditions, and conduct analyses such as parametric studies to validate the solutions;
2. Solve engineering problems requiring minimisation, integration and analysis of functions of several variables;
3. Use Fourier series to solve engineering problems.

Differential calculus of functions of several variables; Integral calculus of several variables, including integral representation of functions and parameter-dependent integrals; Fourier series and their application to civil engineering problems; Ordinary differential equations

## STUDY LOAD

Type	Hours	Percentage
Hours small group	15,0	6.67
Guided activities	9,0	4.00
Hours large group	45,0	20.00
Self study	126,0	56.00
Hours medium group	30,0	13.33

**Total learning time:** 225 h

## CONTENTS

### Topic 1. Metric spaces. Topology

**Description:**

Topology. Basics

Difficulties in consolidating the concepts 1.5h assessment test 0.5h

**Full-or-part-time:** 7h 11m

Theory classes: 2h

Practical classes: 1h

Self study : 4h 11m

### Topic 1. Metric spaces. Topology

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Practical classes: 1h

Self study : 4h 11m



### Item 2. Numerical successions and series

**Description:**

Covergent sequences. Subsequences. Cauchy sequences. Sequences of real numbers. Calculation of limits. Infinitesimals and infinities. Series of real numbers. Absolute convergence. Numerical series of positive terms. Convergence criteria. Power series. Radius of convergence.

Problems sequences and series

Test

**Full-or-part-time:** 24h

Theory classes: 6h

Practical classes: 3h

Laboratory classes: 1h

Self study : 14h

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### Item 3. Differential calculus of real functions of real variable

**Description:**

Definition and concept of derivative. Differentiability and continuity. Chain rule. Derivative of inverse functions. Monotony. Rolle's theorem. Cauchy's theorem. Mean value theorem. Hôpital rule. Higher order derivatives. Taylor polynomials. Taylor's theorem. Local and absolute maxima and minima.

Achievement of the concepts

Test

**Full-or-part-time:** 19h 12m

Theory classes: 4h

Practical classes: 3h

Laboratory classes: 1h

Self study : 11h 12m



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Achievement of the concepts

Test

**Full-or-part-time:** 19h 12m

Theory classes: 4h

Practical classes: 3h

Laboratory classes: 1h

Self study : 11h 12m

### Item 4. Differential calculus of functions of several variables

**Description:**

Limits and continuity of functions of several variables. Partial and directional derivative. The differential of a function. Sufficient condition for differentiability. Chain rule. Inverse and implicit function theorems. Higher order derivatives. Schwarz's theorem. Taylor's theorem. Local and conditional extrema.

Consolidation of concepts

Test

**Full-or-part-time:** 57h 35m

Theory classes: 13h

Practical classes: 7h

Laboratory classes: 4h

Self study : 33h 35m

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Consolidation of concepts

Test

**Full-or-part-time:** 57h 35m

Theory classes: 13h

Practical classes: 7h

Laboratory classes: 4h

Self study : 33h 35m



### Item 5. Riemann

**Description:**

Upper and lower integral. Definition of Riemann integral. Properties. Riemann-integrable functions. Lebesgue theorem. Primitive concept. The integral function. Integration and differentiation. Fundamental theorem of calculus. Barrow's rule. Mean value theorem. Substitution theorem. Integration by parts. Improper integrals. Applications of the integral.

Problems

Test

**Full-or-part-time:** 16h 48m

Theory classes: 4h

Practical classes: 2h

Laboratory classes: 1h

Self study : 9h 48m

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Problems

Test

**Full-or-part-time:** 16h 48m

Theory classes: 4h

Practical classes: 2h

Laboratory classes: 1h

Self study : 9h 48m

### Item 6. Riemann integral multiple

**Description:**

Upper and lower integral. Multiple Riemann integral. Lebesgue criterion. Repeated one-dimensional integration. Fubini's theorem. Jordan measurable sets. Change of variables theorem. Integral representation of functions. Integrals depending on a parameter.

Problems

Test

**Full-or-part-time:** 24h

Theory classes: 6h

Practical classes: 3h

Laboratory classes: 1h

Self study : 14h



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Problems

Test

**Full-or-part-time:** 24h

Theory classes: 6h

Practical classes: 3h

Laboratory classes: 1h

Self study : 14h

### Item 7. Sequences and Functional Series. Fourier series

**Description:**

Sequences and series of functions. Taylor series. Fourier series.

Problems

Consolidation of concepts

**Full-or-part-time:** 14h 23m

Theory classes: 3h

Practical classes: 2h

Laboratory classes: 1h

Self study : 8h 23m

### Item 7. Sequences and Functional Series. Fourier series

**Description:**

Sequences and series of functions. Taylor series. Fourier series.

Problems

Consolidation of concepts

**Full-or-part-time:** 14h 23m

Theory classes: 3h

Practical classes: 2h

Laboratory classes: 1h

Self study : 8h 23m

### Item 8. Ordinary Differential Equations

**Description:**

Introduction and fundamental theorems. Differential equations and modelling. Elementary methods of solution. Linear differential equations. Overall results. Linear differential equations with constant coefficients. Systems of linear differential equations.

Problems. Solving differential equations and systems of ODEs.

Laboratories resolution of ODEs and systems

Test

**Full-or-part-time:** 52h 48m

Theory classes: 11h

Practical classes: 6h

Laboratory classes: 5h

Self study : 30h 48m



### Item 8. Ordinary Differential Equations

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Laboratories resolution of ODEs and systems

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Theory classes: 11h

Practical classes: 6h

Laboratory classes: 5h

Self study : 30h 48m

## GRADING SYSTEM

The final mark is a number in the 0 - 10 range, 0 being the lowest possible mark. Pass mark is 5. The final mark  $N_{final}$  is given by

$$N_{final} = 0.4 * N_c + 0.3 * N_{pg1} + 0.3 * N_{pg2}$$

where  $N_c$  is the mark received from continuous assessment in class activities and  $N_{pg1}$  and  $N_{pg2}$  are the mark received in the global exams.

Continuous assessment consists of several activities, either individual or in group, carried out during the academic year (either in or out of the classroom).

The final exam is intended to check the student's knowledge and understanding of the mathematical concepts studied during the course. It contains a number of questions on concepts related to the learning outcomes and a number of application exercises.

Students who have failed the course but have been passing regularly the continuous assessment tests at the end of each block are eligible for a reassessment exam, which will take place on the date announced in the academic calendar. The highest final mark attainable in the case of reassessment is the pass mark 5.

Students who cannot be assessed due to justifiable absence from lectures or exams can sit the reassessment exam only by special permission of the Head of Studies. In this case, the limitation of 5 on the highest attainable mark will not apply to the blocks assessed for the first time.

Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation tests will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

These tests must be authorized by the corresponding Head of Studies, at the request of the professor responsible for the course, and will be carried out within the corresponding academic period.

## EXAMINATION RULES.

Failure to sit a continuous assessment exam or to hand in an assignment before its deadline will result in a mark of 0 in this exam or assignment.



## BIBLIOGRAPHY

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### Basic:

- Estela, M.R.; Saà, J. Cálculo con soporte interactivo en Moodle. 2008. Madrid: Pearson Educación, 2008. ISBN 978-84-832-2480-9.
- Edwards, C.H.; Penney, D.E. Ecuaciones diferenciales y problemas con valores en la frontera: cómputo y modelado. 4a ed. México: Prentice Hall Hispanoamericana, 2009. ISBN 978-970-26-1285-8.
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- Tenebaum, M.; Pollard, H. Ordinary differential equations: an elementary textbook for students of mathematics, engineering, and the sciences. New York: Dover Publications, 1985. ISBN 0486649407.

### Complementary:

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- Larson, R.; Hostetler, R.; Edwards, B. Cálculo, Vol I, Vol II. 9a ed. México: Mc. Graw Hill, 2010. ISBN 9786071502735.
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- Marsden, J.E.; Weinstein, A. Calculus (3 volums). 2nd ed. New York: Springer-Verlag, 1985. ISBN 0387909745 (V.1) ; 0383909753 (V.2) ; 0387909850 (V.3).
- Bronson, R.; Costa, G.B. Differential equations. 4th ed. New York: McGraw-Hill Education, 2014. ISBN 9780071824859.