Course guides
250301 - ÀLGEBRA - Algebra

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Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: BACHELOR'S DEGREE IN GEOLOGICAL ENGINEERING (Syllabus 2010). (Compulsory subject).

Academic year: 2018 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: FRANCISCO JAVIER MARCOTE ORDAX

Others: FRANCISCO JAVIER MARCOTE ORDAX

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
4048. Ability to solve the types of mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods; numerical algorithms; statistics and optimisation

Transversal:
591. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
597. EFFECTIVE USE OF INFORMATI0N RESOURCES - Level 1. Identifying information needs. Using collections, premises and services that are available for designing and executing simple searches that are suited to the topic.
600. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

TEACHING METHODOLOGY

The course consists of 4 hours a week of classes in the classroom (large group). They are devoted to lectures about 2 hours in large group, in which the teacher explains the concepts of matter and basic materials, and presents examples. A problem-solving exercises and the teacher is engaged approximately 1.5 hours (on average). For each subject, one hour is dedicated to the students meet in class (large group) specific problem, assisted by the teacher. Used material support in the form of detailed teaching plan using the virtual campus ATENEA: content, scheduling of learning and assessment activities conducted and literature.

LEARNING OBJECTIVES OF THE SUBJECT

Students will acquire a general understanding of linear algebra, methods for solving linear problems encountered in engineering, and aspects of analytical geometry. They will also learn to apply this knowledge to specific scientific and technical problems and to geological engineering in general.

Upon completion of the course, students will be able to: 1. Interpret vector spaces;
2. Solve linear equation systems manually and using basic software, and apply the concepts of vector calculus to geometric problems;
3. Perform vector and matrix calculus, solve linear eigenvalue problems manually and using basic software, and use tensors.

Logic, set theory and algebraic structures; Vector spaces, including matrix algebra; Systems of linear equations, the basic algorithms used to solve them, linear applications and bilinear forms; Euclidean spaces; Determinants and their applications, in particular for calculating areas and volumes; Analytical geometry; Linear operators: Endomorphisms and spectral theorems, affine Euclidean spaces, eigenvalues and eigenvectors; Tensor algebra: Basic operations, change of base and tensor calculus

Knowledge of linear algebra, methods of solving linear problems of its own engineering, elements of analytic geometry and ability to apply them in scientific-technological materials and geological engineering in general.

At the end of the course students will have acquired the ability to:
1. Translate and manipulate using matrices (linear) problems that can be formalized with the help of
a linear map.
2. Solving problems of eigenvalues, mainly diagonalization of real or complex matrices.
Recognize the applicability of the diagonalization in different physical or mathematical problems.
3. Dealing with real Euclidean spaces. Treating certain geometric problems from an
algebraic point of view, using the canonical scalar product in R^n.
4. Solving problems of analytic geometry.

Knowledge of' linear maps, space dual, and diagonalization of endomorphisms and matrices; eigenvalues and eigenvectors. Knowledge of billinear forms and Euclidean space. Knowledge of calculating areas and volumes. Knowledge of linear operators: endomorphisms and spectral theorems. Knowledge of analytical geometry, affine spaces, Euclidean affine spaces.

STUDY LOAD

Type	Hours	Percentage
Hours small group	14,0	9.33
Hours large group	26,0	17.33
Hours medium group	20,0	13.33
Self study	84,0	56.00
Guided activities	6,0	4.00

Total learning time: 150 h

CONTENTS

Topic 1: Linear Maps

Description:
Linear maps: definition and examples. The vector space of linear maps. Subspaces kernel and image. Injectivity and completeness. Composition of linear maps. Linear finite-dimensional spaces. A linear range. Fundamental theorem of dimensions. Associated with a linear array: definition, examples, calculating the image of any vector. Matriu de la composició d'aplicacions lineals; matriu associada a un isomorfisme.
Change the basic problem: approach to the problem, resolution and sample matrix. The dual space of a vector space: Dual basis, change of base.
Problems Practice 1.
Problems Practice 1.
Resolution (supervised) problems.

Specific objectives:
Linking with new concepts they already know basic algebra: vector spaces, matrices, systems of linear equations, etc.. Get to know the matrix associated with a linear finite-dimensional spaces and get to know her all the relevant information of the application. Learn bases for subspaces of kernel and image from the array associated with a linear and determine if this deduction is injective, exhautiva, bijective.
Understand that associated with a linear array can be expressed in different bases, and knowledge base changes. Understand the elements of the dual space of a vector space are linear, but also vectors of a vector space, and to laprendre relationship between corresponding arrays. Learn relate matricialment base with its dual basis, learn to get from the other. Able to obtain the components of a vector space dual to some basis, using the basic laws change if necessary.
Practice criteria to follow to determine if an application is not linear. Learn how to get associated with a linear array, and deduce the kernel and image.
Learn to find the inverse of a linear isomorphism in finite dimension. Mastering and understanding the basic process of change. Be familiar with the dual space of a vector space, and know how to find the components of a vector in a dual basis. Learn to find another base which is dual-homed.

Full-or-part-time: 14h 23m
Theory classes: 2h
Practical classes: 2h
Laboratory classes: 2h
Self study : 8h 23m

Topic 2: Reduction of endomorphisms

Description:
Vectors and eigenvalues of an endomorphism. Own subspace associated with an eigenvalue. Finite dimension. Maximum number of linearly independent eigenvectors of an endomorphism. Characteristic polynomial, solutions and features multiplicities; Teormea Fundamentals of Algebra. Upper bound for the dimension of a subspace itself. Endomorphisms diagonalitzables; regarding the existence of a base space consisting of eigenvectors. General Diagonalization theorem.
Problems of Practice 2.
Elementary Diagonalization theorem. Triangulation of endomorphisms; Triangulation Theorem. Triangulation and diagonalization of square matrices. P-th power of a Skippy diagonalitzable. Cayley-Hamilton; applications to calculate the inverse of a matrix or inversible isomorphism, and p-th power endomorphism or a square matrix.
Practice 2

Specific objectives:
Become familiar with the concepts of value and eigenvector of an endomorphism. Learn to find, in finite dimension, all the solutions and features the multiplicities of an endomorphism, knowing distinction between feature and solution value. Get to know their own bases of subspaces of an endomorphism. Learn diagonalitza determine whether or not an endomorphism, obtaining a base matrix is diagonal if so associda.
Learn all solutions and features an endomorphism of the multiplicities in finite dimension. Get to know their own bases of subspaces of an endomorphism. Learn diagonalitza determine whether or not an endomorphism, obtaining a base matrix is diagonal if so associda.
Learn to use the diagonalitzabilitat of a square matrix (or endomorphism) to solve certain types of problems, such as the calculation of the r-th power or calculating the nth term of a sequence r (the Fibonacci type) . Understanding the usefulness of the Cayley-Hamilton theorem to calculate the r-th power of an array that is not diagonalitzable.
Learn to calculate the r-th power of a square matrix diagonalitzable. Knowing the different types of problems can be solved by the diagonalization of a matrix, with emphasis on finding the nth term r of the Fibonacci sequence or other similar sequences. See an example of how to find the r-th power of an array (if not diagonalitzable), using the Cayley-Hamilton.

Full-or-part-time: 28h 47m
Theory classes: 5h
Practical classes: 4h
Laboratory classes: 3h
Self study : 16h 47m

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Topic 3: Bilinear forms

Description:
Bilinear forms on real vector spaces. Symmetric bilinear form. Symmetric bilinear form defined. Finite dimension. Matrix associated to a bilinear form. Calculation of the matrix image of a pair of vectors to a bilinear form. Base change problem. Relationship between symmetry of the array and associated character of a symmetric bilinear form.
Real symmetric matrices: congruence with diagonal matrices. Real symmetric matrices defined. Methods to determine whether a real symmetric matrix is not defined, and what type. Relationship between bilinear forms defined and symmetric matrices (associated) symmetrically defined. Canonical and normal forms of a symmetric bilinear form.
Problems on Practice 3
Problems on Practice 3

Specific objectives:
Learn how to find out if your application is not bilinear form. Get to know the matrix associated to a bilinear form in finite dimension, and know whether the shape is not symmetrical
Learning to diagonalitzar a real symmetric matrix by applying elementary row operations and column (congruence with a diagonal matrix). Learn different methods to determine whether a real symmetric matrix is not defined (and what type if so)
Knowing how to find out if a certain application is not bilinear form. Learn to find the matrix associated to a certain bilinear form. Learn where to find the basis matrix associated with a bilinear form is diagonal. Knowing the different methods to determine whether a matrix bilinear form is symmetric definidida, and what type. Sesquilineals ways to solve problems (complex) type similar to those solved by bilinear forms (actual).

Full-or-part-time: 16h 48m
Theory classes: 4h
Practical classes: 2h
Laboratory classes: 1h
Self study : 9h 48m

Topic 4: Inner product

Description:
Inner product, Euclidean spaces. Norm of a vector. Properties of the norm of Cauchy-Schwarz inequality and triangular. System of orthogonal vectors. Pythagorean theorem Pythagorean theorem and generalized. Unit vector; orthonormal system of vectors. Orthogonal and orthonormal bases of a finite-dimensional subspace.
Methods for finding orthogonal and orthonormal bases of a finite-dimensional subspace: Gram-Schmidt and Gauss-Lagrange; Gram matrix of a system of vectors. Orthogonal subspace, properties and implicit equations in finite-dimensional Euclidean space. Orthogonal projection of a vector on a subspace; interpretation as a "best approximation", and calculation methods in finite dimensional Euclidean spaces.
Problems on Practice 4
Algebraic geometric interpretation of results. The cross product in a three-dimensional real Euclidean space. Mixed product of three vectors. Properties of vector product and mixed product. Calculating area and volume of a paral.lelogram paral.lelepíped.
Problems on Practice 4
Session 19-a
Resolution (supervised) problems

Specific objectives:
Be familiar with the basics, seeing that all the dot product "typical" (in the canonical R3) is only a particular case. Emphasized that all calculations related to different concepts depending on which dot product is used.
Learn how to find and otogonals orthonormal bases of a subspace, by different methods. Knowing and practicing the various methods described to compute the orthogonal projection of a vector on a subspace. Knowing the existence of approximate methods (such as "least squares") operation which is based on the concept of orthogonal projection.
Learn determine whether or not certain applications is dot product. In Euclidean spaces of finite dimension calculated using the scalar product matrix associated with the scalar product. Learn how to get the orthogonal subspace of a given subspace, finite dimensional Euclidean spaces.
Starting with the geometric language translation of some algebraic results on R ^ 2 or R ^ 3 (with the canonical inner product) such as the Pythagorean theorem, understand that the link with the geometry may allow us to interpret geométricamente-so form closer to the intuition-algebraic concepts and many other results. Understand the product of two vectors depends on what the dot product used, and how to calculate vector products (taking into account the orientation of the orthonormal basis of reference). Recognizing the "typical" products and mixed vector as individual cases of treatment that is becoming more generic environment of a three-dimensional real Euclidean space.
In finite dimensional Euclidean space to learn to obtain orthonormal bases and orthogonal subspace of a vector, by different methods.
In finite dimensional Euclidean space to learn to calculate the orthogonal projection of a vector on a subspace, which previously analyzing method may be useful in each case. Algèbricament solve a problem of classical geometry (plan oal'espai). Learn to calculate the vector product and the product mixed in different real Euclidean space R ^ 3.

Full-or-part-time: 36h
Theory classes: 6h
Practical classes: 5h
Laboratory classes: 4h
Self study : 21h

Topic 5: Operators

Description:
Operators in finite dimensional Euclidean spaces. Transpose operator. Properties of transposed operator. Operators Normal basic properties. Spectral theorem for normal operators (in orthonormal basis diagonalization). Some real normal operators: symmetric, symmetric, orthogonal, property type and features of the solutions.
Spectral theorem for symmetric operators. Normal matrices; connection with normal operators. Orthogonal diagonalization Diagonalization of a matrix. Spectral theorem for real symmetric matrices.
Problems on Practice 5
Problems on Practice 5

Specific objectives:
Understand that the transposed of an operator depends on the scalar product used. Knowing the spectral theorem for normal operators know and apply it, finding an orthonormal basis where the operator matrix takes a diagonal from.
Learn symmetric operators are the only real normal which can take a diagonal form in some orthonormal basis. Learn to diagonalize orthogonally an square matrix whenever possible.
Learn how to find the attachment or a transposed operator. Learn determine whether an operator is not normal. Learn to apply the spectral theorem for normal operators, finding an orthonormal basis where the operator diagonalitza.
Learn determine whether a normal operator is one of the types described in the theoretical sessions, and know if it will be deducted diagonalitzable orthonormal basis in the values and how they will be (if so). Learn deduce properties (especially on-spectral-values) for normal operators that do not belong to any of the categories introduced in the theoretical sessions, using techniques similar to those seen in algebraic theory.

Full-or-part-time: 21h 36m
Theory classes: 3h
Practical classes: 4h
Laboratory classes: 2h
Self study : 12h 36m

Topic 6: Affine spaces

Description:
Affine space. Linear varieties; dimension. Equality varieties. Linear variety generated by a set of points. Parallel linear varieties. Intersection and sum of linear varieties. Relative positions of two varieties.
Baricentric coordinates, centroid. Equations of a variety baricentric coordinates. Cartesian coordinates, change of coordinates. Equations in cartesian coordinates of a variety. Affine euclidean space. Orthogonal linear varieties. Distance between two linear varieties.
Isometries. Displacements; classification.
Problems on Practice 6

Specific objectives:
Knowing the different types and ways to express a linear variety, mainly in R ^ 2 and R ^ 3. Learn to find the relative position of two linear varieties.
Learning to find the centroid of a set of points and obtain the equations of a variety in baricentric coordinates. Learn Cartesian coordinate changes. Learn to calculate the distance between two linear varieties.
Understand the concepts of isometry and travel, knowing the different types of displacement when the affine space is Euclidean dimension of 1, 2 or 3.

Full-or-part-time: 19h 12m
Theory classes: 5h
Practical classes: 2h
Laboratory classes: 1h
Self study : 11h 12m

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Synthesis exam

Full-or-part-time: 7h 11m
Laboratory classes: 3h
Self study : 4h 11m

GRADING SYSTEM

- The Final Mark (10 points) is obtained by applying the following formula:

Final Mark = (0.3 *NAC) +(0.2* NA12) +(0.2 *NA34) +(0.3*NPS),

where NAC is a mark which corresponds to classroom activities,
and NA12, NA34 and NPS are the marks (10 points) obtained with the realization of 3 continuous assessment activities, which are described below:

NA12: mark obtained by solving an exam (A12) of Topics 1 and 2, for which you will have 1 hour and 50 minutes for resolution. This exam has two parts: a Control (problem solving) and a Test
(10 questions with 4 possible answers each, with only one correct; each right answer is worth +1 point, each incorrect answer subtract 0.2 points, and each blank answer is worth
0 points). The NA12 score is calculated using the formula:
NA12 = (0.75 *NC12) + (0.25 *NT12)
where NC12 and NT12 are the marks (10 points) obtained with the Control and the Test, respectively. If the score of the test is negative, NT12 = 0 will be taken.

NA34: mark obtained by solving an exam (A34) of Topics 3 and 4. The description of the xiexam is the same as for A12. Score NA34 is calculated using the formula:
NA34 = (0.75 *NC34) + (0.25*NT34)
where NC34 and NT34 are the scores (10 points) obtained with the Control and the Test of Topics 3 and 4, respectively. If the socre of the test is negative, take NT34 = 0.

NPS: mark obtained by solving a Synthesis Exam (PS) on the entire course (duration: 3 h).

- Each student may voluntarily repeat one of the controls (either of the Topics 1 and 2, or of Topics 3 and 4), at a date to be determined, having 50 min. for resolution. If the mark thus obtained exceeds the obtained initially, either the score NA12 or NA34 will be recalculated accordingly.

- Students who do not perform all the 3 activities of continuous assessment (A12, A34, PS) will have a final grade of "No Presentat".

- Other activities can be proposed (voluntary) during the course, without impact on the Final Mark of the course.

Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation tests will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

These tests must be authorized by the corresponding Head of Studies, at the request of the professor responsible for the course, and will be carried out within the corresponding academic period.

EXAMINATION RULES.

- If some activity of continuous assessment is not done in the scheduled period, it will be assigned a score of zero.

- To carry out the exams in class (A12, A34, PS) it is forbidden to use calculators, mobile phones, notes, books or any other device (electronic or otherwise) that allows storing or view information on the subject and / or manipulating mathematical expressions. As an exception, for Synthesis Exam (PS) you can carry an A4 sheet where you can write all the information which you consider may be useful for carrying out the exam.

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Basic:
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- Hoffman, K.; Kunze, R. Álgebra lineal. México D.F.: Prentice-Hall, 1973. ISBN 9688800090.
- Castellet, M.; Llerena, I. Álgebra lineal y geometría. Barcelona: Reverté, 1994. ISBN 8429150099.
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- Burgos, J. Álgebra lineal y geometría cartesiana. 3a ed. Madrid: McGraw-Hill, 2006. ISBN 8448149009.
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