# Course guide
# 320167 - ERM - Elasticity and Strength of Materials

**Last modified:** 19/04/2023

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| **Unit in charge:** | Terrassa School of Industrial, Aerospace and Audiovisual Engineering |
| **Teaching unit:** | 712 - EM - Department of Mechanical Engineering. |

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| **Degree:** | BACHELOR'S DEGREE IN INDUSTRIAL DESIGN AND PRODUCT DEVELOPMENT ENGINEERING (Syllabus 2010). (Compulsory subject). |

**Academic year:** 2023     **ECTS Credits:** 6.0     **Languages:** Catalan

## LECTURER

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| **Coordinating lecturer:** | Javier Alvarez del Castillo |
| **Others:** | Albert Catalan<br>Javier Freire |

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

**Specific:**
CED04-DIDP. Knowledge of elasticity and strength of materials and their application to problem solving in the field of engineering. (Common module for the industrial branch).

## TEACHING METHODOLOGY

Individual study.
Project based cooperative learning, which focuses on problem solving and projects evaluated together.
In theoretical sessions, the professor will explain the theoretical basis of the material, concepts, and methods and illustrate them with appropriate examples to facilitate understanding.
Practical work sessions consist of statements and guided processes to achieve a result.
Students must study independently to assimilate concepts and solving exercises.

## LEARNING OBJECTIVES OF THE SUBJECT

Facilitate and enhance the capacity of abstraction.
Develop and exercise the space imagination.
Introduce concepts, techniques and methodologies in the area of Graphic Expression in Engineering.
Learn the techniques of representation in object design (2D sketches and 3D parametric CAD techniques and representations).

## STUDY LOAD

| Type | Hours | Percentage |
|---|---|---|
| Hours medium group | 15,0 | 10.00 |
| Self study | 90,0 | 60.00 |
| Hours large group | 45,0 | 30.00 |

**Total learning time:** 150 h

# CONTENTS

## TOPIC 1: INTRODUCTION TO ELASTICITY AND STRENGTH OF MATERIALS

**Description:**
1.1 Introduction to elasticity.
1.2 Rational mechanics, strength of materials and theory of elasticity.
1.3 Rigid and deformable bodies.
1.4 Equilibrium of deformable bodies. Fundamental postulates.
1.5 Superposition principle.

**Specific objectives:**
In this topic, we explore the concept of matter by relating it to rational mechanics, and in doing so highlight the continuity and progress of scientific knowledge. We also cover the fundamental postulates on which the equilibrium of deformable systems is based.

**Full-or-part-time:** 4h
Theory classes: 1h
Self study : 3h

## TOPIC 2: STATE OF STRESS

**Description:**
2.1 Concept of stress at a point.
2.2 Stress vector. Intrinsic components.
2.3 State of stress in the neighbourhood of a point.
2.4 Stress tensor.
2.5 Equilibrium equations at internal and external points.
2.6 Calculation of the intrinsic components of a stress vector.
2.7 Stresses and principal axes.
2.8 Invariants of the stress tensor.
2.9 Octahedral stresses.
2.10 Decomposition of stress tensors: spherical and deviator tensors.
2.11 Lamé's ellipsoid.
2.12 Two-dimensional representation of a stress tensor. Mohr's circles. Maximum shear stress.
2.13 Applied exercises.

**Specific objectives:**
This topic explores the concept of stress state. We begin by covering the concept of stress at a point, the stress vector associated with the orientation of a plane in the neighbourhood of a point, and the stress tensor as a general expression of the state of stress in the neighbourhood of a point, from which any stress vector associated with any particular orientation can be found and defined in terms of the stress vectors associated with the three Cartesian coordinate planes. We then cover the relationship between the stress tensor and external forces, derived from the required equilibrium between the internal and external points of the body. We discuss the calculation of the intrinsic components of stress vectors (normal and shear), of stresses and principal axes, of invariants of the stress tensor, of octahedral stresses, and of the decomposition of spherical stress tensors and stress deviator tensors. We then cover the two-dimensional (Mohr's circle) and three-dimensional (Lamé's ellipsoid) representations of the state of stress in the neighbourhood of a point, as opposed to the mathematical representation of stress (stress tensor), highlighting the large amount of information conveyed by these representations.
The following exercises will be carried out as part of this topic: reflection on the concept of equilibrium between the internal and external points of a body in equilibrium that involve calculating the state of stress; applied exercises involving the calculation of the Cartesian and intrinsic components of a stress vector for a given plane orientation in the neighbourhood of a point, whether by taking the stress tensor in said neighbourhood or, failing that, by using external forces in cases with simple geometries and loads; reflection on the principle of reciprocity; the calculation of stresses and principal axes; and various applications of Mohr's circles to three-dimensional stress states.

**Full-or-part-time:** 8h
Theory classes: 2h
Practical classes: 2h
Self study : 4h

## TOPIC 3: STATE OF STRAIN

**Description:**
3.1 Infinitesimal transformation in the neighbourhood of a point. Statement of the transformation. Assumptions made.
3.2 Components of the displacement: translation, rotation and pure deformation.
3.3 Strain tensor.
3.4 Unitary strain vector in any direction. Unit elongation. Slip.
3.5 Angular deformation. Angular distortion.
3.6 Physical interpretation of the components of the strain tensor.
3.7 Principal strains and directions.
3.8 Invariants of the strain tensor.
3.9 Decomposition of strain tensors: spherical and deviator tensors.
3.10 Octahedral strain.
3.11 Strain quadrics.
3.12 Two-dimensional representation of a strain tensor.
3.13 Compatibility conditions for strain tensors.
3.14 Calculation of displacement.
3.15 Applied exercises.

**Specific objectives:**
In this topic, we explore the concept of state of strain in parallel to the state of stress, highlighting the fact that both are defined in terms of their respective tensors, both of which are second-order symmetric tensors, such that the mathematical treatment is the same for both states. We then focus on the physical interpretation of the components of the two tensors. We finish by studying the compatibility conditions for strain tensor components, with a focus on their physical significance and the calculation of displacement.

It is essential that students gain a thorough understanding of the knowledge imparted in these first three topics, both from a mathematical point of view and, especially, in terms of the physical significance of each of the concepts covered. Only with this knowledge will they be able to follow the rest of the subject.

The following exercises will be carried out as part of this topic: applied exercises involving the calculation of strain state given the vector of displacement in the neighbourhood of a point, as well as reflection on the associated concepts of translation, rotation and deformation; the calculation of displacements using strain tensors and applying compatibility equations; exercises involving the calculation of pure strain vectors, longitudinal unit elongation, slip and angular deformations in a given direction in the neighbourhood of a point; and the calculation of principal strains and directions and applications of Mohr's circle.

**Full-or-part-time:** 8h
Theory classes: 2h
Practical classes: 2h
Self study : 4h

## TOPIC 4: RELATIONSHIP BETWEEN STRESS AND STRAIN

**Description:**
4.1 Experimental study of the relationship between stress and strain.
4.2 Simple tensile test. Stress-strain curve. Hooke's law. Young's modulus. Poisson's ratio.
4.3 Generalised Hooke's law. Lamé's equations.
4.4 Applied exercises.

**Specific objectives:**
This topic highlights the need to use experimentation to gain an understanding of the laws governing the behaviour of materials, i.e. the laws of the relationship between stress and strain. We explain the simple tensile test and the data obtained from it. We also introduce Hooke's law, define Young's modulus and Poisson's ratio, and present the generalised Hooke's law and Lamé's equations, which together comprise all of the equations governing the linear elastic behaviour of deformable bodies. Finally, we introduce types of behaviour other than linear elastic behaviour.
The following exercises will be carried out as part of this topic: reflection on the meaning of Young's modulus, the shear modulus, Poisson's ratio, yield strength and fracture; and exercises involving the calculation of strain state using stress state and material properties.

**Full-or-part-time:** 6h
Theory classes: 1h 30m
Practical classes: 1h 30m
Self study : 3h

## TOPIC 5: WORK-ENERGY THEOREMS

**Description:**
5.1 Concept of internal energy or strain energy.
5.2 Relationship between external forces and the corresponding strain. Influence coefficients.
5.3 Expressions of internal energy.
5.4 Maxwell¿Betti reciprocal work theorem.
5.5 Castigliano's theorems.
5.6 Menabrea's theorem.
5.7 Principle of virtual work.
5.8 Applied exercises.

**Specific objectives:**
In this topic, we cover the energy theorems, exploring the various expressions of internal energy and studying the Maxwell¿Betti theorem, Castigliano's theorem and Menabrea's theorem, as well as the principle of virtual work. These theorems are especially relevant in strength of materials applications.
The following exercises will be carried out as part of this topic: reflection exercises involving the calculation of strain energy in bodies deformed under external loading; exercises involving the application of the Maxwell¿Betti reciprocal work theorem, Castigliano's theorem, Menabrea's theorem and the virtual work theorem in order to calculate displacement in cases with loads and geometries that are manageable at this level of the course.

**Full-or-part-time:** 8h
Theory classes: 2h
Practical classes: 2h
Self study : 4h

## TOPIC 6: FAILURE CRITERIA

**Description:**
6.1 Elastic failure criteria. General information.
6.2 Equivalent stress.
6.3 Maximum principal stress criterion.
6.4 Maximum longitudinal strain criterion.
6.5 Maximum shear stress criterion.
6.6 Maximum strain energy criterion.
6.7 Maximum distortion energy criterion.
6.8 Octahedral shear stress criterion.
6.9 Caquot's intrinsic curve theory.
6.10 Mohr¿Coulomb theory.
6.11 Concept of factor of safety. Allowable stresses.
6.12 Applied exercises.

**Specific objectives:**
This topic explores the various elastic failure criteria, providing students with the necessary background to know which criterion to consider for a given type of material. We introduce the concepts of equivalent stress for multiaxial states, allowable stress and factor of safety. This topic is included here because students, having covered the theory of elasticity, should become familiar with the theory's range of applicability in multiaxial cases.
As part of this topic, students will complete exercises that combine the calculation of material properties, geometry, factor of safety, and states of stress or of other external forces, all through simple designs of mechanical elements.

**Full-or-part-time:** 8h
Theory classes: 2h
Practical classes: 2h
Self study : 4h

## TOPIC 7: PRISMATIC MEMBERS

**Description:**
7.1 Purpose and utility of strength of materials.
7.2 Prismatic members: concept and types.
7.3 External forces.
7.3.1 Directly applied forces.
7.3.2 Interfacial adhesion strengths.
7.4 Statically determinate and statically indeterminate systems.
7.5 Assumptions made.
7.6 Limitations of beam theory.
7.7 Calculation process.
7.8 Applied exercises.

**Specific objectives:**
This topic serves as an introduction to strength of materials, and is also the first of several topics covering the basics of prismatic members. We explore various concepts related to prismatic members, including their various types, interfacial adhesion, and the concept of static determinacy and indeterminacy. We emphasise the importance of the starting hypothesis in relation to prismatic members and explore the limitations of prismatic members. Finally, we study the calculation process followed when one works with prismatic members.
The exercises included in this topic could be considered a review of the statics topics covered in the subject Mechanics and Theory of Mechanisms (I). However, they place special emphasis on the calculation of reactions with various types of supports under different cases of loads and geometries, as well as on the calculation of the degree of static indeterminacy.

**Full-or-part-time:** 7h
Theory classes: 4h
Self study : 3h

## TOPIC 8: STRESS ON CROSS-SECTIONS

**Description:**
8.1 Stress on a cross-section of a prismatic member.
8.2 State of stress in a cross-section. Equivalence equations.
8.3 Laws of stress and corresponding diagrams.
8.4 Special case: a prismatic member subjected to a load on its middle plane.
8.5 Equilibrium of cross-sections.
8.6 Applied exercises.

**Specific objectives:**
In this topic, we explore the various kinds of stress that can act on a cross-section of a prismatic member¿ axial force, shear force, bending moment and torsional moment¿and learn how to determine each of them when all external forces are known, establishing the relevant laws and diagrams for statically determinate systems. We will also study the equilibrium of cross-sections.
As part of this topic, students will complete exercises involving the calculation and representation of free-body diagrams (axial force, bending moment, torsional moment, shear force) for prismatic members with different geometries, types of loads (forces and moments with different orientations, distributed and uniform loads) and types of support. Special attention will be given to methodology, sign convention and the study of stresses on cross-sections in equilibrium. Through exercises involving free-body diagrams, we will also explore the relationship between shear force and the slope of the bending moment diagram.

**Full-or-part-time:** 8h
Theory classes: 2h
Practical classes: 2h
Self study : 4h

## TOPIC 10: STRESS AND STRAIN CAUSED BY NORMAL FORCE

**Description:**
10.1 Stress and strain in a cross-section subjected to a normal force. Strain energy.
10.2 Application to the calculation of straight members.
10.3 Stress and strain in a bar under its own weight.
10.4 Bodies of equivalent tensile strength.
10.5 Statically indeterminate systems consisting of elements performing simple compression or tension.
10.6 Erection stress and thermal stress.
10.7 Applied exercises.

**Specific objectives:**
In this topic, we study the normal stress state in cross-sections of prismatic members. We analyse the normal stresses produced by axial force and bending moment. We introduce the concept of the neutral axis, as well as the expression of the relative motion of the faces of an elementary cross-section, starting with the assumption of conservation of plane sections or the Navier¿Bernoulli hypothesis and applying Hooke's law. We study the statement of the generalised Navier¿Bernoulli principle (for cases in which bending occurs along with shear force or torsional moment). Students begin to familiarise themselves with the general case, combined bending, symmetrical bending and special cases of each. Discussion of this subject will continue in Topics 5 and 6.
The exercises for this topic will enable students to:
· Understand the assumptions made in the calculation of normal stresses in cross-sections of prismatic members and, therefore, the inherent limitations of the mathematical model.
· Calculate normal stresses in cross-sections of prismatic members subjected to bending moment and normal force, and understand the reasons behind the mathematical calculation.
· Comprehend the superposition principle as applied to stresses (bending moment and normal force, in this case) as well as its impact on the stress distribution along a cross-section.
· Understand stress distribution on the basis of the effect that stress has on a cross-section and identify the fibres experiencing zero stress (neutral axis), the relative maxima, the area under tension and the area under compression.
· Reflect on and calculate the relative motion of the faces of a differential cross-section, fibre elongation and unit strain in particular directions, and strain energy in a cross-section subjected to stress, as a means of laying the groundwork for the calculation of elastic strain and deformation, as well as the practical use of energy methods.
· Understand the relationships between external stress, unit strain and elongation, as well as the expressions for calculating them.
· Understand the superposition principle as applied to stresses and their effects, and reflect on and calculate cases of combined stress, such as general combined bending, symmetrical bending, and special cases of each.

**Full-or-part-time:** 8h
Theory classes: 2h
Practical classes: 2h
Self study : 4h

## TOPIC 10: STRESS AND STRAIN CAUSED BY NORMAL FORCE

**Description:**
11.1 Symmetric pure bending.
11.1.1 Navier's formula.
11.1.2 Neutral axis.
11.1.3 Section design and modulus.
11.1.4 Deformation of cross-sections.
11.1.5 Geometric efficiency of a section.
11.2 Biaxial bending.
11.2.1 Stress.
11.2.2 Neutral axis.
11.2.3 Section design.
11.2.4 Deformation of cross-sections.
11.3 Applied exercises.

**Full-or-part-time:** 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

## TOPIC 11: STRESS AND STRAIN CAUSED BY BENDING FORCE

**Full-or-part-time:** 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

## TOPIC 12: COMBINED BENDING

**Full-or-part-time:** 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

## TOPIC 13: SIMPLE BENDING: STRESS AND STRAIN CAUSED BY SHEAR FORCE

**Full-or-part-time:** 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

## TOPIC 14: TORSION

**Full-or-part-time:** 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

## GRADING SYSTEM

- 1st exam (elasticity final) : 40 %
- 2nd exam (material resistance final): 40 %
- works: 20 %

All those students who suspend, want to improve note or can not attend the partial exam, will have the opportunity to examine the same day of the final exam. If the circumstances do not make it feasible that it is the same day as the final exam, the teacher responsible for the subject will propose, via the Atenea platform, that the aforementioned recovery test will be carried out another day, during class hours .

The new note of the recovery exam will replace the old one only in case it is higher.

For those students who meet the requirements and submit to the reevaluation examination, the grade of the reevaluation exam will replace the grades of all the on-site written evaluation acts (tests, midterm and final exams) and the grades obtained during the course for lab practices, works, projects and presentations will be kept.

If the final grade after reevaluation is lower than 5.0, it will replace the initial one only if it is higher. If the final grade after reevaluation is greater or equal to 5.0, the final grade of the subject will be pass 5.0.

## BIBLIOGRAPHY

**Basic:**

- Mase, G. E. Mecánica del medio continuo. México: McGraw-Hill, 1977. ISBN 9684512759.
- Solaguren-Beascoa Fernández, Manuel. Elasticidad y resistencia de materiales. Madrid: Pirámide, 2016. ISBN 9788436836042.
- Malvern, L. E. Introduction to the mechanics of a continuous medium. Englewood Cliffs, NJ: Prentice-Hall, 1969. ISBN 9780134876030.