Academic ceremony to award Prof. Alessio Figalli an honorary doctoral degree of the UPC

Laudatio DHC. Professor Alessio Figalli, UPC, 2019

by Xavier Cabré (UPC and ICREA)

Dear Rector, members of the Executive Council, UPC school directors.
Dear Professor Figalli.

I am extremely pleased that my university is conferring the Doctorate Honoris Causa to Alessio Figalli, following an initiative of the dean of the Facultat de Matemàtiques i Estadística. I am pleased not just because of Figalli's relevance within Mathematics, but also because of my scientific relationship with him and his important mentoring of former UPC students. It is an honour for me to introduce him to you by briefly explaining his trajectory and contributions.

Alessio Figalli is a 35-year-old Italian mathematician. He was born in Rome in 1984. From 2002 to 2006, he was an undergraduate student of Mathematics at the Scuola Normale Superiore di Pisa in Italy. He obtained his PhD in 2007, at the age of 23, under the supervision of two great mathematicians: at the Scuola Normale, Luigi Ambrosio (Fermat Prize and Balzan Prize) and at the École Normale Supérieure de Lyon, Cédric Villani (Fields medalist). Within Mathematics, his area of research is Analysis—more precisely, Partial Differential Equations and the Calculus of Variations. I will briefly talk about these areas at the end of my speech.

After his PhD, Figalli spent two years in France with positions at the Université de Nice and the École Polytechnique. He then moved to The University of Texas at Austin, where he has spent most of his career, from 2009 to 2016; he started as associate professor and Harrington Faculty Fellow and later became full professor and holder of the R. L. Moore Chair. Since 2016 he has been full professor and holder of a chair at ETH Zurich in Switzerland.

He has mentored about 10 PhD students and 10 postdocs. He is currently editor of nine mathematical journals. He has published about 150 articles (a very large amount for a mathematician of his age) with about 90 collaborators. I admire a particular feature of his scientific production: the incredibly large spectrum of topics and areas in which he publishes at the top level.

In 2012, Figalli received an important prize: the European Mathematical Society Prize for young researchers aged under 35. In 2018, a year ago, he got the top distinction in math. He won the Fields Medal, which was given to him at the International Congress of Mathematics in Rio de Janeiro. This is the most important prize for mathematicians, together with the more recent Abel Prize. Winners of the Fields Medal must be under 40 years of age; Alessio Figalli got it at 35 and became the second Italian mathematician to get it—Enrico Bombieri was the first, in 1974.

Italy has produced some of the world's best mathematicians since the fifteenth century, among them Cavalieri, Tartaglia, Torricelli, Riccati, Ricci, Ruffini, Beltrami, and Levi, a mathematician whose results I admire a lot. Beppo Levi was Jewish and had to emigrate to Rosario (Argentina) in the early 1930s; there, he joined a group that included Lluís Santaló, the most prominent Catalan mathematician ever. The group succeeded in making Argentina a strong country in Mathematics in the twentieth century. In fact, Lluís Santaló was awarded an honorary doctoral degree by the UPC in 1977. Taking into account that of Michael Atiyah in 2008, we are now celebrating the third honorary doctoral degree awarded to a mathematician at our university.

More recently, many of the top Italian mathematicians have studied at the Scuola Normale Superiore di Pisa, like Figalli. This shows the great importance of having institutions that focus on excellence and on the best young students—a system largely implemented in countries like France or the USA, for example, but much less so in Spain. In this respect, the Facultat de Matemàtiques I Estadística (FME) and the Centre de Formació Interdisciplinar Superior (CFIS) of the UPC can be proud to have produced some of the best young mathematicians in Spain in recent decades. However, in my opinion, if our country wishes to have a Fields medalist at some point, an even stronger focus on top students must be put, even at the undergraduate level.

In recent years, several FME students have won the Prize José Luis Rubio de Francia from the Real Sociedad Matemática Española, which is awarded every year to a young mathematician working in Spain. It is one of the most important prizes in Mathematics in Spain. Xavier Ros-Oton and Joaquim Serra, who both took their degree and PhD at the UPC, are some of the recent awardees. Alessio Figalli is responsible for this success. Indeed, he mentored Xavier and Joaquim as postdoctoral researchers, in Austin (Texas) and Zurich. This has had a huge impact on their growth as professional mathematicians. All three (Alessio, Joaquim and Xavier) now live in Zurich and work together very intensely. In addition to what they have all already produced in joint works, I know that we will soon be receiving further great scientific news from them.

Xavier Fernández-Real is another great student who took his undergraduate degree at the UPC. He moved to Austin to do his PhD under the supervision of Figalli, and he is now completing it in Zurich. Xavier has already published eight research articles and could have finished his PhD; it is not easy, however, to give up on all that one can learn if one is being trained by Alessio Figalli. We can be very pleased that Fernández-Real will add to the list of outstanding mathematicians who took their degree at the UPC. For all this, thanks, Alessio! The impact on the future of Catalan Mathematics will be considerable.

Let me now explain how I first met Alessio Figalli. The story bears some relation to Ennio De Giorgi (1928–1996), a professor at the Scuola Normale and one of the most important mathematicians of the twentieth century. Having finished my PhD, I started work on a famous conjecture by De Giorgi. A conjecture is a statement that some mathematicians believe to be true. Still, until a complete mathematical proof of it is found, the statement could turn out to be false. While I was trying to prove the De Giorgi conjecture, in 1995, I had the chance to spend three months as a European Union postdoc at the Scuola Normale and I met De Giorgi in person. I was impressed—back then, I was only 29! Meeting him strengthened my will to solve the conjecture. I started working on it with Luigi Ambrosio, a former student of De Giorgi's and future PhD supervisor to Figalli. Four years later, in 1999, Ambrosio and I could solve part of the conjecture. Our article became one of the main topics in Figalli's *tesi di laurea*, his undergraduate thesis. I remember receiving several e-mails from "student Alessio Figalli". The e-mails clearly showed maturity. I was impressed when he pointed out that in our paper one mathematical claim needed some more justification. It was a very subtle point, difficult to detect by an undergraduate student! We were all happy finding out that there was a quick and precise justification of the claim.

The following is a much more recent interaction and, I believe, a beautiful story. After my three months in Pisa in 1995, I moved to Paris for a postdoc. There I worked with an important French mathematician, Haim Brezis. At that time, he was very interested in a problem that captured my attention. The problem turned out to be very difficult to solve and became a conjecture—we could call it the Brezis conjecture. I worked on it from then on and published several partial developments. In 2013 I made important progress, but that still did not solve the conjecture. Three years ago I went to Zurich and explained the conjecture and progress on it to Alessio Figalli. He was captivated by its beauty and we worked together intensively for a week, but still could not solve it. After that, I discussed the problem with other several very famous specialists in the field, at several top institutions, again without success. A year ago I returned to Zurich for a great conference that Alessio Figalli, Joaquim Serra, and Xavier Ros-Oton organised at ETH. I asked them to meet at ETH, this time all four of us, to give the Brezis conjecture one more try. In three days, we proved it to be correct; we found a proof for it! Without them, it would not have been solved yet. Therefore, many thanks! It has become a beautiful joint article by the four of us, which we finished last July, and to me it represents the culmination of 23 years of work on the problem.

As a result, I am lucky enough to have been doing math with Alessio Figalli intensively over the last year. I have got to know him better, mathematically. He has all the aptitudes that a mathematician needs: curiosity, depth, sense of beauty, technical skills, ambition, optimism, and perseverance. Several times I have been astonished by how far and deep he can get, and how quickly, from a good idea. He also has a very useful quality: an appropriate measure of being practical, being efficient. This makes me think of something I read (on the occasion of his Fields Medal) about his childhood: he would do all his homework first, to avoid parents complaints and to be able to fully enjoy playing with other kids afterwards! This is efficiency. Doing math with him is a joy—one just keeps learning and learning. At the same time, as a person he is easy, nice, friendly, uncomplicated. He just makes things simple, and with no pain.

Later on, Professor Figalli will present some details about his area of research. I will therefore be brief, but I would in any case like to say a few words about it. First of all, we should be aware that almost all scientific and technological issues in modern life have some Mathematics behind them. Second, very often, and still today, such Mathematics are highly non-trivial, or are simply not available yet and several decades, at least, may be needed for them to be developed, to be understood. This requires the work of mathematicians themselves, but also their interaction with physicists, engineers, chemists, etc. In addition to these disciplines (traditionally more mathematically oriented), others such as Biology, Sociology, Financial investing, Big data and the World Wide Web are demanding more new Mathematics. We have lots of work to do, and we like it!

Alessio Figalli's field of research is Partial Differential Equations, traditionally called the equations of Mathematical Physics. The most famous ones are the heat equation (think of a reactor, or of ice melting in water), the wave equation (think of an auditorium, or of a Wi-Fi network, or of how to find out, through waves, whether there is oil underground) and the equations of fluids (think of ocean streams, or of weather prediction). It is usually impossible to write the exact solution to these equations for a concrete body or situation. It is thus of great importance to learn more (qualitatively and quantitatively) about the essential properties of their solutions. In Partial Differential Equations, this requires the use of many sophisticated mathematical tools. Once and only once their essential properties are understood will we be able to compute the solutions in concrete situations through numerical methods, with the aid of computers.

Note that one often needs more than three variables to describe three-dimensional objects. For instance, Newton's equations to describe the movement of one planet involve six dimensions (three positions and three velocities) plus time. The Brezis conjecture that we solved last year claimed that, up to nine dimensions, the observable stationary solutions of certain heat equations are always regular, that is, nice, smooth, without singularities, and hence easier to compute. Since 1975 we had known that this fact was not true for 10 dimensions or more—note that this was 44 years ago.

The 2018 Fields Medal citation for Alessio Figalli reads: "For contributions to the theory of optimal transport and its applications in Partial Differential Equations, Metric Geometry and Probability". Indeed, Figalli discovered that the theory of optimal transport has enormous applications to presumably unrelated phenomena, such as to some equations in Mathematical Physics, for instance, those modelling liquid crystal screens, to mention but one. He will explain to us what optimal transport, or optimal allocation of resources, is, as formalised by the French mathematician Gaspard Monge in 1781. Let me just introduce it with an example: think of a big hole in a garden and a pile of exactly the right amount of soil to fill the hole. I am responsible for choosing which portions of soil are to be moved to which portions of the hole, knowing that this is going to be an exhausting physical task and I do not want to be too tired at the end of my day of gardening. I may initially try to save energy by moving the soil closest to the hole to the part of the hole that is closest to the soil; this will, however, make me work harder later on, moving the farthest soil to the farthest parts of the hole. The problem is therefore what the most economical way of transporting the soil is, in the sense of which part of the soil must be brought to each part of the hole to minimise the total amount of work done. It turns out that this is an extremely rich mathematical problem that took several centuries to be understood. It was only fully solved and understood a few years ago.

Let me mention three of Alessio Figalli's most relevant contributions. The first is on the regularity of transport maps. Figalli and Guido de Philippis (*Inventiones Mathematicae*, 2013) got a striking result that had remained open since Luis Caffarelli's fundamental results in the nineties. It establishes regularity for the Monge-Ampère equation (which is behind the optimal transport map) when the right-hand side is just measurable. A second breakthrough contribution is in the theory of free boundary problems; here, for ice melting in water, one might wonder how regular the contact surface between ice and water, which evolves over time, is. In a paper of Figalli's, written in collaboration with Joaquim Serra (*Inventiones Mathematicae*, 2019), the authors make a breakthrough and largely improve Caffarelli's seminal results from the eighties. A third area in which Figalli's contributions have been fundamental, from 2009 onwards, concerns stability issues in geometric inequalities. Since ancient Greece, we have known that, among regions with equal perimeter, the circle is the one that has the greatest area inside. Quantitative versions of this fact are of great interest. That is, one might like to know how close to a disk a region is, knowing that the relation between perimeter and area is very close to the optimal one given by disks. Figalli's results here are numerous and a real revolution in this fundamental area.

Finally, though there are many more, let me mention his contributions to the theory of the spectra of random matrices using transport techniques, as well as to weak KAM theory in Hamiltonian systems—a topic of interest for our Dynamical Systems group at the UPC.

To finish, let me thank Professor Figalli for accepting the UPC's honorary doctoral degree. This is an honour for our university. I also thank you all for your attention and hope that you enjoy this ceremony as much as I am doing.

Acceptance speech by Prof. Alessio Figalli

Dear Rector, dear members of the Executive Council,
Dear professors, dear students, dear staff,
Dear ladies and gentlemen,

It is a great honour for me to receive an honorary doctoral degree from the Universitat Politècnica de Catalunya.

Besides feeling extremely honoured to receive an honorary doctoral degree from a university as prestigious as the UPC, you should know that several mathematicians from the UPC had and still have a very important role in my career, and this makes such an award even more special to me.

I would like to start with a short story. When I was just a student, in Pisa in 2004, I had to write my bachelor's thesis and I ended up doing it on a problem called the De Giorgi conjecture. De Giorgi was a very famous Italian mathematician, among the best mathematicians of the last century, and he was particularly famous not just for his mathematical results but also because he was enormously intuitive, which led him to put forward many conjectures. He conjectured results in mathematics that he believed to be true, and people would spend many years trying to prove what De Giorgi had conjectured.

In 1978, De Giorgi stated a famous conjecture that was related to two very important areas of mathematics. On the one hand, it was related to minimal surfaces. Minimal surfaces are surfaces that minimise the area, so we see them, for instance, when we take a metal wire and we look at the soap film that bounds the wire. This will be a minimal surface, that is, a surface that has the least possible area among all surfaces that bound a certain curve. On the other hand, his conjecture involved partial differential equations (PDEs). PDEs are a very important area of mathematics and they appear in physics, chemistry, biology, etc. Almost all physical phenomena are described by PDEs.

The De Giorgi conjecture is still open today. However, there have been some serious fundamental results that answered the De Giorgi conjecture in a series of important cases. The goal of my bachelor's thesis was to understand the statement of the conjecture and read some of the results by advanced mathematicians on this topic. I still remember that one of the main references for my thesis was a very important paper written jointly by my advisor at the time, Luigi Ambrosio from Pisa, and Xavier Cabré, a professor here at the UPC.

This work was the beginning of my interaction with Professor Cabré: while reading his paper I had many questions, so I sent him several e-mails and, even though I was just a student, he very kindly replied to all of my questions. I was fortunate enough to meet him later on in my career. I still remember when, in 2008, I held a position in France and I was considering moving to UT Austin, in Texas. In November 2008, I went to UT Austin as a visitor, and Xavier Cabré also happened to be visiting at that time. Cabré had been a professor in Austin a few years before he moved back to Barcelona. Deciding whether or not to move to the USA was not an easy decision, and I was extremely lucky to have lots of discussions and interaction with Cabré. He encouraged me to go to Austin, and that was fundamental to my research and my career, so I'm very grateful to him for his support and the advice he gave me.

To continue, in 2014 Cabré had two very strong PhD students at the UPC. One of them was Xavier Ros-Oton, who decided to embark on a postdoc in Austin and work with me. This was the beginning of a long collaboration that still continues today, as Xavier Ros-Oton works at the University of Zurich, so that was another interaction with a mathematician from the UPC. Cabré's other student came to Zurich as a postdoc in 2016. This was Joaquim Serra. And just to finish, another student from the UPC, a bachelor's student, who then started his PhD with me and was suggested to me by Cabré; this is Xavier Fernández-Real. As you can see, I have had many interactions with mathematicians from the UPC, and they are very visible on my list of collaborators.

To conclude this part, I have more good news to share with you about my interaction with the UPC's mathematicians: today's story started in 2004 with Cabré and the De Giorgi conjecture, and now I'd like to continue with another conjecture called the Brezis conjecture. More than 20 years ago, Haim Brezis, a French-Israeli mathematician, made a very important conjecture on PDEs. I'm very happy to announce that, just three months ago, with Cabré, Ros-Oton and Serra, we solved it! Such a good end to the story!

This story shows why receiving an honorary doctoral degree from the UPC is so special to me. It is a celebration of all my interactions with this university, and also a celebration of mathematics. Mathematics is a dynamic subject that has enormous applications in everyday life. People do not often recognise it, but in fact mathematics is present everywhere, in all the technology that we use, in all the tools that we have. Just to mention a few: recording devices, computers, artificial intelligence, cryptography; the list goes on and on...

Concerning mathematics, in my career I have worked on many problems, but one that I'm attached to and that played an important role in me being awarded the Fields Medal is optimal transport. The origin of optimal transport dates back to the end of the eighteenth century and Gaspard Monge, a French mathematician who worked in the Napoleonic era. Monge's goal was to find the cheapest possible way of transporting material from one place to another; let's say from the mines where the material was extracted to the construction sites where the fortifications had to be built. This was a challenging problem that fascinated Monge, and he made several important contributions to it.

After Monge, the theory did not move on for many years, until the 1940s, roughly 150 years later, when Leonid Kantorovich, a Russian mathematician and economist, studied it again. He made a very important contribution to the topic, and he did in fact receive the Nobel Prize in Economics for his contribution in 1975.

Then, in the 1980s, mathematicians started to work more and more on optimal transport, and this was the beginning of the modern mathematical study of the problem. In particular, in addition to its obvious applications in economics, what fascinated mathematicians was the (surprising) discovery that optimal transport could be used as a tool for understanding other topics.

From my perspective, I was extremely fascinated by two applications of optimal transport. One application concerns the study of crystals. The rough idea is that the shape of a crystal is determined by its molecular structure, and there are physical principles that tell us why a crystal has a particular shape. In particular, one can show that the shape of a crystal comes from minimising some sort of surface tension.

However, while idealisation is important, in real life crystals are subject to a series of external factors (gravity, temperature, external forces, etc.). Therefore, the question we might like to know the answer to is how the presence of these external factors influences the shape of a crystal. In other words, we might think of a crystal in a room and the room's temperature starting to rise. The shape of the crystal starts to change because of the added heat that one puts into the system, and we would like to estimate the change in shape in terms of the extra amount of energy added to the system.

It is a particularly impressive fact that we can use optimal transport to study this problem! The rough idea is that via optimal transport we can understand how the molecules of which the crystal is composed move in the process of heating, and we can use this tool to estimate the change in shape with optimal bounds! This is something I studied approximately 10 years ago.

Another thing that I've been working on is the relation between optimal transport and meteorology. We have realised that some systems in meteorology called semi-geostrophic equations have an optimal transport principle behind them. Here the idea is that clouds are made of particles that move in time. Then, by looking at the overall movement of the cloud, we might want to understand how each individual particle is involved in this general movement. Again, one of the principles behind it is optimal transport: each particle moves from an instant of time to the next instant of time, minimising a transportation cost. Through this principle, my collaborators and I have managed to solve the semi-geostrophic system equations.

These two examples are particularly important for me because they are personal, but I would like to mention that optimal transport actually goes beyond these applications. For instance, optimal transport has recently been applied in image processing: given two images, assume that one wants to transport colours from one image to another. Then, since optimal transport is a tool that allows us to move material by thinking of images as distributions of pixels, one can use it to transport pixels from one picture to another.

In addition, more recently people have been using optimal transport to compare different images (for instance, to decide whether an image represents the photo of a cat or a dog), and this plays a crucial role in machine learning, the aim of which is to train computers to recognise such images.

In recent years, mathematics has been progressing very quickly, and the more society advances, the more mathematics needs to progress, and vice versa, the more mathematics advances, the more technology we are able to develop.

Although mathematics is usually split into two areas, applied and pure mathematics, these two areas progress together, one thanks to the other, and their advancement is crucial for the development of other sciences and technology in our society.

Today is therefore not just a celebration of my research, not just a celebration of my honorary doctoral degree, but I think of it as a celebration of the world of mathematics.

I sincerely thank the Universitat Politècnica de Catalunya for honouring me with this honorary doctoral degree.