Course guide
200001 - CV - Single Variable Calculus

Last modified: 17/05/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 7.5 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: RAFAEL RAMIREZ ROS

Others:

Primer quadrimestre:
MARIA ALBERICH CARRAMIÑANA - ANIVE
SEBASTIA MARTIN MOLLEVI - M-A
SARA MATHEU MARTINEZ DEL CAMPO - M-A, M-B
RAFAEL RAMIREZ ROS - M-A, M-B
JORDI VILLANUEVA CASTELLTORT - M-B

Segon quadrimestre:
SEBASTIA MARTIN MOLLEVI - REF
RAFAEL RAMIREZ ROS - REF

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

The teaching of the course will be divided into two separate blocks: theory and problems. In the theory sessions we will develop the theoretical content of the course, based on the different results and demonstrations. In addition, we will include examples to consolidate the concepts introduced. At problem sessions we will combine theoretical and complicated exercises so that students get a maximum depth level in the field of mathematical analysis of a variable, with more mechanical ones that students must master, such as the calculation of limits and integration. Various continuous evaluation activities will be carried out consisting of in-person tests and/or problem submissions (in theory classes) and/or virtual tests at flexible times.

LEARNING OBJECTIVES OF THE SUBJECT

The main objetive of this course is to make the student familiar to the basic concepts of calculus on one variable. The fundamentals of calculus that are needed in the other subjects of the degree are provided. The sutudents are introduced to deduction techniques in calculus and more generally, to proof methods in an axiomatic system.

STUDY LOAD

Type	Hours	Percentage
Guided activities	7,5	4.00
Hours small group	30,0	16.00
Hours large group	45,0	24.00
Self study	105,0	56.00

Total learning time: 187.5 h

CONTENTS

Sequences of real numbers

Description:
Axiomatic introduction to real numbers. Basic topology in R. Defintion of sequences. Bounded sequences. Monotone sequences. Limit of a sequence. Convergent sequences. Partial sequences. Cauchy sequences. Different definitions of real numbers. Bolzano-Weierstrass theorem. Infinite limits. Computation of limits. Introduction to numerical series, for example the harmonic serie and the geometric serie.

Full-or-part-time: 35h
Theory classes: 8h
Practical classes: 6h
Self study : 21h

Real variable functions. Limits.

Description:
Functions. Basic definitions. Limit in a point. Characterization by sequences. Lateral limits. Enlarging the concept of limit: infinite limit and limit in the infinite. Infinites and infinitesimals. Computation of limits. Introductions to the elementary functions: exponential, trigonometric, hyperbolic,...

Full-or-part-time: 22h 30m
Theory classes: 5h
Laboratory classes: 4h
Self study : 13h 30m

Real variable functions. Continuity.

Description:
Pointwise continuity. Types of discontinuities. Continuous functions. Propieties. Theorems on continuous functions. Uniform continuity. Heine theorem.

Full-or-part-time: 20h
Theory classes: 5h
Practical classes: 3h
Self study : 12h

Real variable functions. Differentiability.

Description:
Pointwise differentiation. Tangent line. Differentiability and continuity. Differentiation rules. Higher order derivatives. Implicit differentiation. Theorems on differentiable functions. Local approximation: Taylor theorem and consequences. Maxima and minima. Optimization.

Full-or-part-time: 45h
Theory classes: 11h
Laboratory classes: 7h
Self study : 27h

Integrable functions. Riemann integral.

Description:
Antiderivatives. Computation of antiderivatives. Techniques of integration: by parts, by substitution. Integration of rational functions. Integration of trigonometric functions. The lower and upper integral. Definition of Riemann integral. Propieties. Riemann integrable functions. Integration and continuity. Integration and differentiation. Fundamental theorem of calculus. The definite integral and antiderivatives: Barrow's rule. Mean value theorem. Applications.

Full-or-part-time: 32h 30m
Theory classes: 8h
Laboratory classes: 5h
Self study : 19h 30m

GRADING SYSTEM

The grading is based on three items:
1. Continuous evaluation (AC).
2. Mid-term exam (EP).
3. Final exam (EF).

Completion of the corresponding block of the course "Ús solvent de la informació" will be a requirement to be graded.

The overall grade (NF) will be computed as follows:
NF=max{0.60*EF+0.30*EP+0.10*AC; 0.70*EF+0.30*EP; 0.90*EF+0.10*AC; EF}

An extra exam will take place on July for students that failed during the regular semester.

BIBLIOGRAPHY

Basic:
- Spivak, Michael. Calculus : càlcul infinitesimal [on line]. 3rd ed. Barcelona: Reverte, 1995 [Consultation: 26/06/2023]. Available on: https://web-p-ebscohost-com.recursos.biblioteca.upc.edu/plink?key=100.65.135.150_8000_572986369&AN=2615591&site=ehost-live&db=nlebk&scope=site. ISBN 8429151370.
- Bartle, R.G. ; Sherbert, D.R. Introducción al análisis matemático de una variable. 2ª ed. Mèxic: Limusa, 1996. ISBN 9681851919.

Complementary:
- Ortega Aramburu, Joaquín M. Introducció a l'anàlisi matemàtica. 2a ed. Bellaterra: Universitat Autònoma de Barcelona, Servei de Publicacions, 2002. ISBN 8449022711.
- Strang, Gilbert; Herman, Edwin. Calculus, vol. I [on line]. Openstax, 2020 [Consultation: 26/06/2023]. Available on: https://d3bxy9euw4e147.cloudfront.net/oscms-prodcms/media/documents/CalculusVolume1-OP.pdf.
- Burgos, Juan de. Cálculo infinitesimal de una variable [on line]. Madrid: Mc Graw Hill, 2007 [Consultation: 26/06/2023]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=3964. ISBN 9788448156343.

RESOURCES

Other resources:
The problem collection "Aprende Cálculo con Youtube" (version 2.0) accessible at https://web.mat.upc.edu/rafael.ramirez/ACcY/