Course guide
200003 - FM - Fundamentals of Mathematics

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
751 - DECA - Department of Civil and Environmental Engineering.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: MARCOS NOY SERRANO

Others: Primer quadrimestre:
MARIA LUZ ALBEROLA PEREZ - M-A
JAUME MARTÍ FARRÉ - M-A, M-B
MARCOS NOY SERRANO - M-A, M-B
MIQUEL ORTEGA SÁNCHEZ COLOMER - M-A

Segon quadrimestre:
JAUME MARTÍ FARRÉ - REF
MARCOS NOY SERRANO - REF

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Theoretical classes essentially consist in instructor presentations, including detailed examples. In practical sessions, some problems are solved by the instructors as a model, and some others by the students.

LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the course is to help saving the bridge between secondary school mathematics and university mathematics by providing students the necessary foundation for developing their undergraduate studies.

This objective involves two intertwined lines. One is to make students aware of the essential role of the concept of proof in mathematics. The other one is to securely establish the basic contents related to language, numerical sets, and elements of algebra.

STUDY LOAD

Type	Hours	Percentage
Guided activities	7,5	4.00
Self study	105,0	56.00
Hours small group	30,0	16.00
Hours large group	45,0	24.00

Total learning time: 187.5 h

CONTENTS

Mathematical formalism: statements and proofs

Description:
Logical propositions. Truth tables. Tautologies and contradictions. Logical equivalence. Expressions with quantifiers. Predicates and variables. Statements and proofs. Proof techniques: implication, equivalences, statements with quantifiers. Induction. Summations and products. Arithmetic and geometric progressions.

Full-or-part-time: 28h 45m
Theory classes: 7h
Practical classes: 5h
Self study : 16h 45m

Sets and mappings

Description:
Set and subset. Inclusion and equality. Power set. Operations: union, intersection, diference, complementary, cartesian product. Correspondence and mapping. Images and antiimages by a mapping. Injective, exhaustive and bijective mappings. Mapping composition. Identity mapping. Mapping inverse.

Full-or-part-time: 28h 45m
Theory classes: 7h
Practical classes: 5h
Self study : 16h 45m

Relations, operations and structures

Description:
Binary relations on a set. Equivalence relations. Equivalence class. Quotient set. Partitions. Canonical decomposition of a mapping. Order relations. Notable elements in a partially ordered set. Algebraic structures: group, ring and field. Ordered field. Boole algebra. The symmetric group. Permutations, cycles and transpositions. Decomposition in cycles and in transpositions. Order and sign of a permutation.

Full-or-part-time: 31h 30m
Theory classes: 9h
Practical classes: 4h
Self study : 18h 30m

Number sets. Numerability

Description:
Equipotent sets. Finite and infinite sets. Cardinal. Number sets: naturals, integers, rational and real numbers. Numerable and enumerable sets.

Full-or-part-time: 16h 45m
Theory classes: 4h
Practical classes: 3h
Self study : 9h 45m

The field of complex numbers

Description:
The field of complex numbers. Real and imaginary parts. The imaginary unit. Ordered pair and binomial form. The conjugate. Module and argument. Trigonometric expression and polar expression. Powers and roots. Exponential of a complex number. Exponential expression of a complex number. Matrix expression of a complex number.

Full-or-part-time: 16h 45m
Theory classes: 4h
Practical classes: 3h
Self study : 9h 45m

Arithmetic

Description:
The ring of integer numbers. Invertible elements. Divisors. The divisibility relation. Theorem of the Euclidean division. Prime numbers. Fundamental Theorem of Arithmetic. Greatest common divisor and lowest common multiple. Bézout identity and Euclid's algorithm. Diophantine equations. Congruences. Congruence relation. The ring of modular integers. Invertible elements and zero divisors. Equations with congruences.

Full-or-part-time: 28h 45m
Theory classes: 7h
Practical classes: 5h
Self study : 16h 45m

Polynomials

Description:
Polynomial with an indeterminate. Equality of polynomials. Algebraic structure. Euclidean division and factorization. Divisors of a polynomial. Prime polynomials. Theorem of factorial decomposition. Greatest common divisor. Euclid's algorithm and B\'ezout identity. Polynomial functions. Roots of a polynomial. Multiplicity of a root. Fundamental Theorem of Algebra. Prime polynomials with comlex, real or rational coefficients. Polynomials with coefficients in Z_p. Rational fractions. Algebraic structure. Simple fractions (complex and real). Decomposition of rational fractions into simple fractions.

Full-or-part-time: 28h 45m
Theory classes: 7h
Practical classes: 5h
Self study : 16h 45m

GRADING SYSTEM

The subject is assessed by means of the continuous assessment and a final exam. The continuous assessment mark will be obtained from a not eliminatory midterm exam (similar to the final exam), and from qualifying some other activities carried out during the term.

The final mark of the subject wil be worked out according to the formula:

Mark = max{final exam mark; 70% final exam mark + 25% midterm exam mark + 5% other activities}.

An extra exam will take place on July for students that failed the regular semester.

BIBLIOGRAPHY

Basic:
- Rosen, Kenneth H. Matemática discreta y sus aplicaciones [on line]. 5a ed. Madrid: McGraw-Hill Interamericana, 2004 [Consultation: 26/06/2023]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=4143. ISBN 9788448140731.
- Bloch, Ethan D. Proofs and fundamentals [on line]. 2nd ed. Boston: Springer Science + Business Media, 2011 [Consultation: 26/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-7127-2. ISBN 0817641114.

Complementary:
- Houston, Kevin. How to think like a mathematician. 1a edició. Cambridge University Press, 2009. ISBN 9780521719780.
- Pla, Josep. Introducció a la metodologia de la matemática. 1ª edició. Barcelona: Publicacions de la Universitat de Barcelona, 2006. ISBN 9788447530656.
- Cunningham, D. W. A Logical introduction to proof. 2013. Springer, ISBN 9781489990990.