Course guide
200101 - FVC - Complex Variable Functions

Last modified: 12/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: ORIOL SERRA ALBO

Others: Segon quadrimestre:
JORDI GUARDIA RUBIES - CFIS, M-A, M-B
PAU MARTIN DE LA TORRE - CFIS, M-A, M-B
ORIOL SERRA ALBO - CFIS, M-A, M-B

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

There are three one hour lectures and two one hour problem sessions per week.

LEARNING OBJECTIVES OF THE SUBJECT

Introduce the holomorphic functions of one complex variable.
Apply the theorem of Cauchy and the winding number to the computation of integrals by residues.
Operate with power series, discuss the radius of convergence and the behaviour on the boundary.
Discuss applications of holomorphic functions.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	24.00
Self study	112,5	60.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

The Complex Plane

Description:
Complex numbers (representation, basic properties, successions, series). The complex plane and its topology.

Full-or-part-time: 7h
Theory classes: 3h
Practical classes: 4h

Holomorphic functions

Description:
Complex variable functions. Derivative. Complex derivative. Cauchy-Riemann conditions. Power series. Holomorphic functions. Examples

Full-or-part-time: 16h
Theory classes: 10h
Practical classes: 6h

Integration. Cauchy Theorem

Description:
Line integrals. Local Cauchy's theorem. Cauchy's integral formula. Zeros of analytic functions. Consequences.

Full-or-part-time: 16h
Theory classes: 10h
Practical classes: 6h

Meromorphic functions. Residue Theorem

Description:
Index of a curve with respect to a point. Homology. Global Cauchy's theorem. Isolated singularities. Laurent series. Residue theorem and applications.

Full-or-part-time: 19h
Theory classes: 11h
Practical classes: 8h

Additional topics: Conformal applications, harmonic functions, z-Roieman function, approximation of meromorphic functions, analytic prolongation.

Description:
Conformal transformations. Riemann's theorem. Schwarz's reflection principle. Harmonic functions. Dirichlet's problem. The z-Riemann function. Theorem of Runge. Analytic prolongation

Full-or-part-time: 17h
Theory classes: 11h
Practical classes: 6h

GRADING SYSTEM

There will be a mid-term exam (ME) and a final exam (FE).
The final grade (NF) will be given by the formula NF = max(FE ; 0.3 * ME + 07 * FE).

An extra exam will take place on July for students that failed during the regular semester.

BIBLIOGRAPHY

Basic:
- Ortega Cerdà, J. Anàlisi complexa [on line]. Barcelona: Universitat Politècnica de Catalunya. Departament de Matemàtica Aplicada I, 1997 [Consultation: 21/06/2023]. Available on: http://hdl.handle.net/2117/189905.
- Stein, E. M. ; Shakarchi, R. Complex analysis. Princeton University Press, 2003. ISBN 0691113858.
- Bruna, J. ; Cufí, J. Anàlisi complexa. Publicacions UAB, 2008. ISBN 9788449025594.
- Ahlfors, L. V. Complex analysis : an introduction to the theory of analytic functions of one complex variable. 3rd. McGraw Hill, 1979. ISBN 0070006571.

Complementary:
- Beck, M.; Marchesi, G.; Pixton, D.; Sabalka, L. A First course in complex analysis [on line]. San Francisco State University, 2009 [Consultation: 21/06/2023]. Available on: https://matthbeck.github.io/complex.html.
- Gamelin, T.W. Complex analysis. Springer, 2001. ISBN 0387950931.
- Rudin, W. Real and complex analysis. 3a ed. McGraw Hill, 1974. ISBN 0070542341.
- Lang, S. Complex analysis. 4th. Springer, 1999. ISBN 0387985921.
- Conway, J. B. Functions of one complex variable. 2nd. Springer, 1978. ISBN 0387944605.