Course guide
200122 - GD - Differential Geometry

Last modified: 17/05/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: JAUME AMOROS TORRENT

Others: Segon quadrimestre:
JAUME AMOROS TORRENT - M-A, M-B
JOSE BURILLO PUIG - M-A
PEDRO TALAVERA SANCHEZ - M-B

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
9. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

(Section not available)

LEARNING OBJECTIVES OF THE SUBJECT

(Section not available)

STUDY LOAD

Type	Hours	Percentage
Hours small group	30,0	16.00
Self study	112,5	60.00
Hours large group	45,0	24.00

Total learning time: 187.5 h

CONTENTS

1. Plane and space curves

Description:
Parametrized curves. Tangent line. Examples. Regular curves, arc length. Curvature, normal vector, binormal vector, torsion, Fermat formulae and Frenet-Serret apparatus. The Fundamental theorem of the theory of curves.

Full-or-part-time: 37h 30m
Theory classes: 9h
Laboratory classes: 6h
Self study : 22h 30m

2. Surfaces

Description:
Regular surfaces and parametrizations. Differentiable functions on surfaces. Critical points. Tangent plane, normal line. Diffential of a map, diffeomorphisms. Geometry in the tangent plane. First fundamental form. Geometry in surfaces. Measure of length, angles and area.

Full-or-part-time: 37h 30m
Theory classes: 9h
Laboratory classes: 6h
Self study : 22h 30m

3. Gauss Curvature

Description:
The Gauss map. Differential of the Gauss map and second fundamental form. Normal curvature and second fundamental form. Normal curvature: Meusnier Theorem. Principal curvatures, curvature lines. Principal curvatures. Lines of curvature. Rodrigues and Euler theorems. Gauss and mean curvature. Classification of points on a surface. Asymptotic curves and direction. Dupin's indicatrix.

Full-or-part-time: 37h 30m
Theory classes: 9h
Laboratory classes: 6h
Self study : 22h 30m

4. Examples of surfaces

Description:
Basic formulas: Weingarten's equations. Flat surfaces. Ruled surfaces. Quadrics. Surfaces of revolution. Minimal surfaces.

Full-or-part-time: 12h 30m
Theory classes: 3h
Laboratory classes: 2h
Self study : 7h 30m

5. Fundamental equations of surface theory

Description:
Isometries and local isometries. Christoffel's symbols. Gauss' Formula and the Theorema Egregium. Codazzi-Mainardi's compatibility equations. Bonnet's theorem.

Full-or-part-time: 25h
Theory classes: 6h
Laboratory classes: 4h
Self study : 15h

6. Geometry on a surface

Description:
Covariant derivative and parallel transport.Geodesics, geodesic curvature, Liouville's formula. The exponential map, minimality properties of geodesics. Sums of the angles of a spherical triangle; Gauss-Bonnet's theorem and applications.

Full-or-part-time: 22h 30m
Theory classes: 5h
Laboratory classes: 4h
Self study : 13h 30m

7. Introduction to differential manifolds

Description:
Differential manifolds, differentiable functions. Tangent space and differential of a function. Regular values, subvarieties. Examples.

Full-or-part-time: 15h
Theory classes: 4h
Laboratory classes: 2h
Self study : 9h

GRADING SYSTEM

The subject mark will be obtained from:

ME: Midterm Exam
PA: Programming Assignment
FE: Final Exam

by the following formula:

Final Mark = max(0.1 PA + 0.9 FE, 0.3 ME + 0.1 PA + 0.6 FE).

An extra exam will take place on July for students that failed during the regular semester.

EXAMINATION RULES.

The exams (ME and FE) will contain theoretical and practical questions.

Only a formulary will be allowed.

BIBLIOGRAPHY

Basic:
- Pascual Gainza, Pere. Geometria diferencial de corbes i superfícies [on line]. Barcelona: Iniciativa Digital Politècnica, 2017 [Consultation: 19/06/2023]. Available on: http://hdl.handle.net/2117/104841. ISBN 9788498806441.
- Shifrin, Theodore. Differential geometry: A First Course in curves and surfaces [on line]. University of Georgia, 2016 [Consultation: 21/06/2023]. Available on: http://alpha.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf.

Complementary:
- Carmo, Manfredo Perdigão do. Differential geometry of curves and surfaces. Englewood Cliffs, NJ: Prentice Hall, 1976. ISBN 0132125897.
- Hitchin, Nigel. Geometry of surfaces [on line]. 2013. University of Oxford, [Consultation: 19/06/2023]. Available on: https://www.e-booksdirectory.com/details.php?ebook=256.
- Bär, Christian. Elementary differential geometry [on line]. Cambridge University Press, 2010 [Consultation: 19/06/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=803056. ISBN 9780521721493.
- Palais, Richard S. A Modern course on curves and surfaces [on line]. Apunts, Brandeis University, 2003 [Consultation: 19/06/2023]. Available on: https://virtualmathmuseum.org/Surface/a/bk/curves_surfaces_palais.pdf.
- Topogonov, Victor Andreevich. Differential geometry of curves and surfaces: A Concise Guide [on line]. Birkhähuser, 2006 [Consultation: 19/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b137116. ISBN 0817643842.
- Milnor, John. Morse theory. Princeton, 1969. ISBN 0691080089.

RESOURCES

Other resources:
*Famous Curves Applet Index http://www-history.mcs.st-and.ac.uk/Java/ />*3D-XplorMath, de Richard Palais: http://3d-xplormath.org/
*Wolfram mathworld curves http://mathworld.wolfram.com/topics/Curves
*National Curve bank http://curvebank.calstatela.edu/home/home.htm />*Open Geometry Gallery http://www1.uni-ak.ac.at/geom/opengeometry_gallery
*Virtual Math museum http://virtualmathmuseum.org/Surface/gallery_o
*Wolfram mathworld surfaces http://mathworld.wolfram.com/topics/Surfaces
*Other gallleries: http://faculty.evansville.edu/ck6/GalleryTwo/Introduction2