Course guide
200151 - ALN - Numerical Linear Algebra

Last modified: 19/05/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: JUAN RAMON PACHA ANDUJAR

Others: Segon quadrimestre:
JUAN RAMON PACHA ANDUJAR - M-A, M-B
ÓSCAR RODRÍGUEZ DEL RÍO - M-A, M-B

PRIOR SKILLS

The student must have achieved the objectives detailed in the teaching guide for the Linear Algebra subject (code AL-200002) which is taught in the first semester of the Mathematics Degree.

REQUIREMENTS

Have followed at least one Linear Algebra course with content similar to the first year Linear Algebra course of the Mathematics Degree. Consult the Teaching Guide, code AL-200002.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

The 5 hours of class per week are divided into 3 hours in a conventional classroom and 2 hours in a computer classroom. Generally, theoretical concepts are presented and developed in the conventional classroom. In the computer classroom, mostly problems, examples of implementation and use of numerical methods, as well as examples of application in science and engineering are done. The progress of the proposed practical exercises is also monitored.

All information regarding the organization and monitoring of the subject, and all teaching material is published on the teaching intranet.

LEARNING OBJECTIVES OF THE SUBJECT

The subject has two main objectives: (1) to give a global idea of the role of numerical methods in solving common problems in mathematics, physics and engineering, and (2) to provide a solid foundation in the numerical solution of linear algebra problems.

The student must acquire abilities to:

- Know and understand the possibilities, and limitations, of numerical methods for solving problems in mathematics and other disciplines

- Know and understand the basic numerical techniques for solving systems of linear equations and eigenvalue problems.

- Select and use an appropriate numerical method for solving a specific problem, identifying its advantages and disadvantages.

- Acquire competence and agility when expressing the numerical methods studied in the form of algorithms to, finally, code them efficiently in the Python programming language.

- Critically analyze the results obtained (precision in the result of interest, adequacy of the numerical method and the mathematical model, interpretation of the results).

- Present the results clearly and concisely.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	24.00
Guided activities	7,5	4.00
Self study	105,0	56.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

Introduction to numerical methods

Description:
- Introduction: Iterations, recurrences, finite differences, etc.
- The Python programming language. Numpy, sympy and matplotlib libraries.

Related competencies :
GM-CE3. CE-3. Have the knowledge of specific programming languages and software.

Full-or-part-time: 17h 30m
Practical classes: 4h
Self study : 13h 30m

Finite arithmetics and accuracy

Description:
- Binary representation of floating point numbers. The IEEE-754 standard for floating-point arithmetic.
- Errors in numerical operations. Propagation of errors. Progressive and regressive analysis of error propagation.
- Numerical stability and instability. examples Conditioning of numerical algorithms.

Related competencies :
GM-CE2. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.

Full-or-part-time: 20h
Theory classes: 5h
Laboratory classes: 3h
Self study : 12h

Linear systems of equations. Direct Methods

Description:
- Basic concepts. Matrix typology. Orthogonality. Vector and matrix rules, and equivalence.
- Gaussian elimination. Partial and total pivots, LU, Cholesky, Doolittle and Crout factorizations.
- QR factorization. Gram-Schmidt orthogonalization and stable variants. Givens and Householer orthogonal projectors.
- Overdetermined systems and least squares approximations.
- Condition number of a matrix and errors in the solution of linear systems. Iterative refinement.
- Band matrices, block and scattered matrices. LU factorization of band and sparse matrices, and fill-in. Incomplete LU factorization, criteria. Use as preconditioners.
- Factorization of partitioned matrices. Substructuring and Schur's complement.

Related competencies :
GM-CE2. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.

Full-or-part-time: 57h 30m
Theory classes: 15h
Laboratory classes: 8h
Self study : 34h 30m

Linear systems of equations: iterative methods

Description:
- Classical stationary methods: Jacobi, Gauss-Seidel, on relaxation (SOR).
- Methods for symmetric matrices. Gradient and conjugate gradient.
- Preconditioners.

Related competencies :
GM-CE2. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.

Full-or-part-time: 35h
Theory classes: 8h
Laboratory classes: 9h
Self study : 18h

Eigenvalues and singular value decomposition, SVD

Description:
- Power method and derivatives: Inverse power, shifted and shifted inverse.
- Jacobi's method. Reduction to tridiagonal form of symmetric matrices.
- Iterations based on QR factorizations. Reduction to Hessenberg form.
- Decomposition into singular values, SVD.
- Applications.

Related competencies :
GM-CE2. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.

Full-or-part-time: 57h 30m
Theory classes: 15h
Laboratory classes: 8h
Self study : 34h 30m

GRADING SYSTEM

The subject is assessed through exams (E) and a set of practices that must be delivered within a specified period (AC). The grade of the subject will be the weighted average

NF = 0.8E + 0.2AC

where the grade E is calculated from the grades of the partial exams (EP), final (EF) com

E = max(0.3 EP + 0.7 EF , EF)

In the extraordinary call E is the minimum between 7 and the mark of the extraordinary exam.

Although only the delivered exercises will be assessed for the practice grade (AC), it is advisable to do all the proposed exercises, as some exam questions may be based on them or require similar technical developments.

Given that part of the exams will consist of the application of the codes developed in the practice classes, the exams will be held in the PC classrooms.

BIBLIOGRAPHY

Basic:
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Complementary:
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