Course guide
200171 - MMF - Mathematical Models in Physics

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 748 - FIS - Department of Physics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: CARLES BATLLE ARNAU

Others: Segon quadrimestre:
CARLES BATLLE ARNAU - M-A
NARCISO ROMAN ROY - M-A

PRIOR SKILLS

The course "Mathematical Models of Physics" is the second general physical content and the first block of matter "modeling" Math Grade FME . This subject is based on the knowledge of the subject of Physics in Q4 and expands its own theoretical formulations of classical mathematical physics using mathematical tools, mainly from multivariable calculus, that the student knows at this point. The course should also provide a base to discuss real systems such as in "Mathematical models of technology" and in different subjects as "Dynamical systems and analysis" and "Numerical methods and engineering."

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-1. Propose, analyze, validate and interpret simple models of real situations, using the mathematical tools most appropriate to the goals to be achieved.
2. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.

Generical:
5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
9. CG-1. Show knowledge and proficiency in the use of mathematical language.
10. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
11. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
12. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
13. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
14. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
15. SUSTAINABILITY AND SOCIAL COMMITMENT. Being aware of and understanding the complexity of social and economic phenomena that characterize the welfare society. Having the ability to relate welfare to globalization and sustainability. Being able to make a balanced use of techniques, technology, the economy and sustainability.
17. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
18. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

The course is designed for a total of 65 teaching hours (13 weeks) in 39 hours of theory sessions and 26 hours of practical sessions (problems). Both in the theoretical and, above all, in the practice sessions, we try to involve the students in their development, inviting them to solve the problems proposed and even to develop a theoretical section.

In the sessions of problems, besides the exercises to be discussed in class, other ones will be delivered to the students. Some of them will be required, and the others could be delivered voluntarily. These exercises would be discussed in the tutorial hours or, exceptionally, in class.

Another objective is to habituate the students to use English bibliography.
Catalan and spanish will be both used in the courses.

LEARNING OBJECTIVES OF THE SUBJECT

The main goal of the course is the familiarization with the core ideas of three fields of classical physics and of quantum mechanics, and their mathematical formulations. The student will adquire the conceptual tools that will allow him to progress in these fields in an autonomous way, and to interact with physicists and engineers.

The detailed learning objectives are:
• To understand the Lagrangian and Hamiltonian formulations of mechanics.
• To use the calculus of variations in order to state the variational principles of mechanics, and their connection with symmetries and conservation laws via Noether's theorem.
• To apply the Lagrangian and Hamiltonian frameworks to discuss complex mechanical problems.
• To describe the Minkowskian formulation of special relativity and the transformations of Lorentz and Poincaré.
• To understant the Lorentz covariance of Maxwell equations.
• To apply the equations of special relativity to solve simple kinematical problems.
• To understand the basic formulation of relativistic dynamics and their application to simple collision problems.
• To understand the several conservation laws of fluids mechanics, both in integral and differential forms.
• To describe Euler and Navier-Stokes equations, and their dominions of validity.
• To understand the historical evolution of quantum mechanics.
• To describe the basic principles of quantum mechanics, and the main differences with respect to classical mechanics.
• To solve simple problems in the framework of quantum mechanics, both with a finite and infinite number of degrees of freedom, mainly in one space dimension.

STUDY LOAD

Type	Hours	Percentage
Self study	112,5	60.00
Hours large group	45,0	24.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

Classic mechanics

Description:
1) Foundations of mechanics. Dynamical systems. Fundamental principles. Galilean invariance. Dynamical systems: configuration and state spaces.
Constraints. Generalized coordinates and velocities.
2) Variational calculus. Three basic problems of the calculus of variations. Hamilton's variational principle. Euler-lagrange equations. Some applications.
3) Lagrangian formalism. Lagrangian systems. Mechanical lagrangians and conservative systems. Constants of motion, symmetries and Noether's theorem.
4) Hamiltonian formalism. Legendre transformation. Generalized moments. Hamiltonian function and Hamilton's equations. Hamilton-Jacobi variational principle.
Hamiltonian systems. Poisson brackets. Constants of motion and conservation laws.

Full-or-part-time: 16h
Theory classes: 10h
Practical classes: 6h

Special relativity

Description:
1) Foundations of special relativity. pre-relativistic classical mechanics and Maxwell equations. Postulates of special relativity. Space-time and Minkowski metrics.
2) Relativistic kinematics and dynamics and electromagnetism. Lorentz and Poincaré transformations.
Relativistic kinematics: time dilatation, length contraction and velocity addition. Relativistic dynamics: quadrimoment.
Covariant form of Maxwell equations: quadripotentials and electromagnetic tensor. Some ideas on general relativity.

Full-or-part-time: 15h
Theory classes: 9h
Practical classes: 6h

The equations of fluid mechanics

Description:
1) Mathematical description of fluid mechanics. Lagrangian and Eulerian coordinates. Material derivative. Transport theorem. Continuity equation. Incompressible fluids. Local deformation of a fluid.
2) The equations of fluid mechanics. Balance of mechanical momentum. Newtonian fluids. Navier-Stokes equations. Balance of mechanical energy. First principle of thermodynamics. Bernoulli's theorem. Balance of entropy.
3) Reynolds' number. Irrotational flow.

Full-or-part-time: 10h
Theory classes: 6h
Practical classes: 4h

Quantum mechanics

Description:
1) Historical and conceptual introduction. Short journey through theoretical physics. The experimental foundations of quantum physics.
2) Quantum mechanics I: wave function. Rules of quantum mechanics in one space dimension. Time-independent Schrödinger equation. Heisenberg's uncertainty principle. Stern-Gerlach experiment. Spin. Harmonic oscillator.
3) Quantum mechanics II: formal postulates. Dirac's notation. Completeness relation. Pure states. Observables. Results of measurements. State reduction. Time evolution. Unitary transformations.

Full-or-part-time: 24h
Theory classes: 15h
Practical classes: 9h

GRADING SYSTEM

At the end of the first two parts of the course there is a first partial exam, with a 45% weight in the final qualification of the subject. After finishing the course, students can choose to perform a second partial exam of the two remaining parts, weighting 45% of the final grade, or a final exam of the entire course, whose value would be, in this case , 90% of the final grade.
The remaining 10% will come from the correction of the problems submitted by the students during the course. An extra exam will take place on July for students that failed during the regular semester.

BIBLIOGRAPHY

Basic:
- Carroll, Sean M. Spacetime and geometry : an introduction to general relativity. San Francisco: Addison Wesley, cop. 2004. ISBN 0805387323.
- Kundu, Pijush K; Cohen, Ira M; Dowling, David R. Fluid mechanics [on line]. 5th ed. Amsterdam: Elsevier, cop. 2012 [Consultation: 01/06/2022]. Available on: https://www-sciencedirect-com.recursos.biblioteca.upc.edu/book/9780123821003/fluid-mechanics. ISBN 0123821010.
- José, Jorge V; Saletan, Eugene J. Classical dynamics : a contemporary approach. Cambridge: Cambridge University Press, 1998. ISBN 0521636361.
- Galindo, Alberto; Pascual, Pedro. Quantum mechanics I. Springer-Verlag, 1990. ISBN 9783642838569.

Complementary:
- Goldstein, Herbert; Safko, Joh; Poole, Charles P. Classical mechanics. 3rd ed. San Francisco: Addison-Wesley, cop. 2002. ISBN 0201657023.
- Garrido Beltrán, Lluís; Pons Ràfols, Josep Maria. Mecànica quàntica. 2a ed. Barcelona: Universitat de Barcelona, 2007. ISBN 9788447532353.
- Feynman, Richard P; Leighton, Robert B; Sands, Matthew L. The Feynman lectures on physics [on line]. New millennium ed. New York: Basic Books, cop. 2010 [Consultation: 26/06/2023]. Available on: https://www.feynmanlectures.caltech.edu/I_toc.html. ISBN 9780465024940.
- Woodhouse, N. M. J. Special relativity. Berlin: Springer-Verlag, cop. 2003. ISBN 9781852334260.
- Jackson, John David. Classical electrodynamics. 3rd ed. New York: John Wiley & Sons, cop. 1999. ISBN 047143132X.

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RESOURCES

Hyperlink:
- Aaronson, S.. Resource

Other resources:
1) S. Aaronson, Introduction to quantum information science, UT course (Austin), 2017. Course materials can be found here: https://www.scottaaronson.com/cs378/esource />
2) Additional resources that will be available in ATENEA.