Course guide
200202 - TOPA - Algebraic Topology

Last modified: 24/05/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

Academic year: 2024 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: JOSEP ALVAREZ MONTANER

Others: Segon quadrimestre:
JOSEP ALVAREZ MONTANER - M-A

PRIOR SKILLS

The abilities acquired in the Topology module.
The abilities acquired in the Affine and Euclidean Geometry module.
The abilities acquired in the Algebraic Structures module.
The user and programmer abilities with diverse software platforms used in several modules for symbolic and numerical calculus and graphical representation will be useful.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generical:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Half of the class time will be devoted to lectures about the theoretical contents of the module, delivered by the lecturer. The other half will be devoted to the discussion and solution of problems related to these contents by the lecturer and the students, and also to the presentation of specific enhancement tasks chosen and developed by the students among the offer made available by the lecturer subject to agreement with her.

LEARNING OBJECTIVES OF THE SUBJECT

* To be acquainted with the most basic homology theories (simplicial and singular) and computing them for a wide range of topological spaces and variants of the theory.
* To be acquainted with several geometrical applications of singular homology.
* To be acquainted with the concept of topological manifold for a general finite dimension, and in this context, with those of local homology, orientations, and with the dimension theorem.
* To be acquainted with the fundamental group as an essential tool to study topological spaces, also by understanding its relation with the first homology group.
* To be acquainted with the computation of the fundamental group for a wide range of topological spaces and versions.
* To understand the intertwinning between different areas of mathematics and , in particular, to understand how topological problems can be solved by algebraic means and viceversa.

STUDY LOAD

Type	Hours	Percentage
Hours large group	30,0	20.00
Self study	90,0	60.00
Hours small group	30,0	20.00

Total learning time: 150 h

CONTENTS

Algebraic Preliminaries

Description:
Finitely generated abelian groups and the classification algorithm.
Abelian groups: complexes.
Homology of complexes.
Homotopies between morphisms of complexes.
Homology long exact sequence.

Full-or-part-time: 10h
Practical classes: 4h
Self study : 6h

Simplicial Homology

Description:
Polyhedra and triangulable spaces.
Simplicial homology of triangulable spaces.
Interpretation of H_0.

Full-or-part-time: 14h
Theory classes: 4h
Practical classes: 4h
Self study : 6h

Singular Homology

Description:
Singular chain complex of a topological space. Singular homology of a topological space.
Continuous maps homotopies.
Homotopical invariance.
Small chains theorem.
Mayer-Vietoris long exact sequence.
The singular homology of the spheres.

Full-or-part-time: 41h
Theory classes: 10h
Practical classes: 9h
Self study : 22h

Topological manifolds

Description:
Local homology.
Dimension invariance.
Orientation.

Full-or-part-time: 17h
Theory classes: 5h
Practical classes: 4h
Self study : 8h

The Fundamental Group and covering spaces

Description:
The fundamental group of a topological space.
Homotopical invariance.
The fundamental group of the circle.
The Seifert-Van Kampen theorem.
The Hurewicz isomorphism.
The covering space of a topological space.
The fundamental group of a covering space
Universal covering and the classification problem

Full-or-part-time: 40h 40m
Theory classes: 10h
Practical classes: 9h
Self study : 21h 40m

GRADING SYSTEM

Work in Problem classes, projects during the term and a final work or examn. The student can request a final exam.
The qualification of the course will be based on the work done by the student in the class of problems, the elaboration of some small project during the course (continuous assessment , up to 60% of the overall mark) , and a final test , which will consist of an exam or the preparation of a project. Students may decide to perform only a final exam.

BIBLIOGRAPHY

Basic:
- Massey, William S. A Basic course in algebraic topology. New York: Springer-Verlag, cop. 1991. ISBN 038797430X.
- Navarro, V.; Pascual, P. Topologia algebraica. Barcelona: Edicions Universitat de Barcelona, 1999. ISBN 8483381230.
- Vick, James W. Homology theory : an introduction to algebraic topology. 2nd ed. New York [etc.]: Springer-Verlag, cop. 1994. ISBN 0387941266.
- Kosniowski, Czes. Topología algebraica. Barcelona: Reverté, cop. 1986. ISBN 8429150986.
- Bott, R.; Tu, L. Differential forms in algebraic topology. ISBN 978144197400.

Complementary:
- Munkres, James R. Elements of algebraic topology. The Benjamin/Cummings Publishing Company, 1984. ISBN 0201045869.
- Hatcher, Allen. Algebraic topology. Cambridge ; New York: Cambridge University Press, 2002. ISBN 0521795400.
- Maunder, Charles Richard Francis. Algebraic topology. Mineola, New York: Dover, 1996. ISBN 0486691314.
- Bredon, Glen E. Topology and geometry. New York [etc.]: Springer-Verlag, cop. 1993. ISBN 0387979263.
- Castellet, Manuel. Introducció a la topologia algebraica. Bellaterra: Universitat Autònoma de Barcelona, Servei de Publicacions, 1994. ISBN 8449002060.
- Brown, Ronald. Topology and groupoids. Deganwy: [s.n.], 2006. ISBN 1419627228.
- Sato, Hajime. Algebraic topology : an intuitive approach. Providence: American Mathematical Society, 1999. ISBN 0821810464.
- Dieck, Tammo Tom. Algebraic topology [on line]. Zürich: European Mathematical Society Publ. House, 2008 [Consultation: 27/06/2023]. Available on: https://web-s-ebscohost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=d11bb5e6-67c1-4762-94ab-a5cc53cb95ee%40redis&vid=0&format=EB. ISBN 9783037190487.
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