

# Course guide

## 200211 - AF - Functional Analysis

Last modified: 01/06/2023

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.  
981 - CRM - Mathematical Research Centre.

**Degree:** BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

**Academic year:** 2023    **ECTS Credits:** 6.0    **Languages:** Catalan

### LECTURER

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**Coordinating lecturer:** ALBERT MAS BLESA

**Others:** Segon quadrimestre:  
RENZO BRUERA MÉNDEZ - M-A  
ALBERT MAS BLESA - M-A

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

8. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.
9. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
10. CE-3. Have the knowledge of specific programming languages and software.
11. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
12. CE-1. Propose, analyze, validate and interpret simple models of real situations, using the mathematical tools most appropriate to the goals to be achieved.

#### Generical:

3. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.
4. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
5. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
6. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
13. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
14. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.

#### Transversal:

1. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
2. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.

## TEACHING METHODOLOGY

**Theory:** the classes will consist of presentations by the professor of definitions, statements, demonstrations and examples. Emphasis on relationships between concepts apparently different for the student will be made.

**Problems:** Presentation of solutions to a collection previously proposed to the student. Also, resolution of some problems by the students themselves.

Among the objectives of the course, more importance will be given to problem solving and to the ability to relate with other areas of mathematics than the mere acquisition of theoretical knowledge.

## LEARNING OBJECTIVES OF THE SUBJECT

Section not available.

## STUDY LOAD

Type	Hours	Percentage
Self study	90,0	60.00
Hours large group	30,0	20.00
Hours small group	30,0	20.00

**Total learning time:** 150 h

## CONTENTS

### Banach Spaces

**Description:**

- Banach spaces
- Examples
- Linear operators and norm
- Fundamental theorems on linear bounded operators (open mapping, closed graph, uniform boundedness)

**Full-or-part-time:** 30h

Theory classes: 6h

Laboratory classes: 6h

Self study : 18h

### Hilbert spaces

**Description:**

- Scalar product
- Projections
- Representation theorems: Riesz-Frechet, Lax-Milgram
- Adjoints in Hilbert spaces
- Orthonormal bases

**Full-or-part-time:** 45h

Theory classes: 9h

Laboratory classes: 9h

Self study : 27h



### Applications

**Description:**

- Motivation; boundary value problems in dimension 1
- Sobolev spaces
- Weak/classical solutions in dimension 1
- Existence/uniqueness and regularity issues

**Full-or-part-time:** 45h

Theory classes: 9h

Laboratory classes: 9h

Self study : 27h

### Compact operators

**Description:**

- Properties
- Spectrum
- Fredholm alternative
- Self-adjoint compact operators

**Full-or-part-time:** 30h

Theory classes: 6h

Laboratory classes: 6h

Self study : 18h

## GRADING SYSTEM

There will be a partial exam, that will determine 30% of the final mark, and a final exam with 50%. The remaining 20% is evaluated from the assignments and expositions in class. The final mark, obtained with these proportions, could be increased, according to the development of the course.

## BIBLIOGRAPHY

**Basic:**

- Rakotoson, Jean-Emile; Rakotoson, Jean-Michel. Analyse fonctionnelle appliquée aux équations aux dérivées partielles. 1999. ISBN 1130498388.
- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan [etc.]: Springer, 2008 [Consultation: 27/06/2023]. Available on : <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pg-origsite=primo&docID=3062992>. ISBN 9788847007512.
- Brézis, H. (Haim). Análisis funcional : teoría y aplicaciones. Madrid: Alianza, 1984. ISBN 8420680885.
- Evans, Lawrence C. Partial differential equations. Providence, Rhode Island: American Mathematical Society, cop. 1998. ISBN 0821807722.

**Complementary:**

- Hirsch, F.; Lacombe, G. Elements of functional analysis. New York [etc.]: Springer, cop. 1999. ISBN 0387985247.
- Stein, E.; Schakarchi, R. Real analysis : measure theory, integration and Hilbert spaces. Princeton Lectures in Mathematics, 2005. ISBN 0691113866.