Course guide
200213 - SD - Dynamical Systems

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: JOAQUIM PUIG SADURNI

Others: Primer quadrimestre:
GEMMA HUGUET CASADES - M-A
JOAQUIM PUIG SADURNI - M-A

PRIOR SKILLS

Basic knowledge about the theory of ordinary differential equations (developed in the course of Differential Equations).

Basic knowledge about the numerical resolution of ordinary differential equations (developed in the course of Numerical Calculus).

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.
13. CE-1. Propose, analyze, validate and interpret simple models of real situations, using the mathematical tools most appropriate to the goals to be achieved.
14. CE-3. Have the knowledge of specific programming languages and software.

Generical:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

The course consists of four hours a week that will include the theoretical and practical aspects of dynamical systems, as well as problem solving and elaboration of individual or group projects.

To encourage the students to learn independently, they will be assigned, during the course, selected problems from the list of problems, small projects during the course and a final synthesis project of the subject. Problems and projects should be presented to other students.

There will be a content review exam at the end of the course where it can appear both theoretical questions and problems similar to those performed in class.

LEARNING OBJECTIVES OF THE SUBJECT

One aims that at the end of the course the student has a set of techniques and results that allow him/her to address the basic aspects of the description and analysis of dynamical systems, whether they are discrete or modeled through differential equations. Additionally, one aims at providing a broad vision of the different lines of applications and research that dynamical systems have (such as celestial mechanics, mathematical biology, neuroscience and epidemiology) and the basic skills for their simulation and quantitative study through computational tools.

STUDY LOAD

Type	Hours	Percentage
Hours small group	30,0	20.00
Hours large group	30,0	20.00
Self study	90,0	60.00

Total learning time: 150 h

CONTENTS

One-dimensional Chaotic Dynamics

Description:
Interval maps Types of orbits. Study of the quadratic family. Symbolic dynamics. Bernoulli shift. Chaos in interval maps. Definitions Lyapunov exponents.

Full-or-part-time: 17h
Theory classes: 4h
Laboratory classes: 4h
Self study : 9h

Lineal Systems

Description:
Linear systems. Classification of linear systems. Non-autonomous linear systems. Stability and conjugation of periodical systems.

Full-or-part-time: 17h
Theory classes: 4h
Laboratory classes: 4h
Self study : 9h

Invariant objects of Flows and Diffeomorphisms

Description:
Critical points of vector fields and fixed points of difeomorphisms. Periodic orbits of fields. Poincaré map. Lyapunov exponents. Hyperbolic points. Stable and unstable manifolds. Conjugation and equivalence. Hartman theorem. Non-hyperbolic points. Center manifold theorem

Full-or-part-time: 59h
Theory classes: 10h
Laboratory classes: 10h
Self study : 39h

Planar Flows

Description:
Planar systems. Poincaré-Bendixson theorem. Liénard Systems. Limit cycles and applications in biology. Introduction to bifurcation theory

Full-or-part-time: 17h
Theory classes: 4h
Laboratory classes: 4h
Self study : 9h

Global Dynamics

Description:
Global invariant manifolds. Homoclinic and heteroclinic points. Smale's Horseshoe. Smale homoclinic theorem. Splitting of separatrices. Method of Poincaré-Melnikov-Arnol'd. Applications to population and epidemiological models, neuroscience and celestial mechanics.

Full-or-part-time: 40h
Theory classes: 8h
Laboratory classes: 8h
Self study : 24h

GRADING SYSTEM

There will be an exam at the end of the course. The grade of the exam will correspond to 20% of the final grade.

The oral presentation and the written resolution of the problems and projects assigned during the course will be evaluated, as well as the participation in classes. This grade will correspond to 60% of the final grade.

The final project execution, the written report and its oral presentation will be evaluated. This grade will correspond to 20% of the final grade.

EXAMINATION RULES.

The assigned problems and projects will be done individually. The final project can be done in groups of up to two people.

BIBLIOGRAPHY

Basic:
- Brauer, Fred; Castillo-Chávez, Carlos. Mathematical models in population biology and epidemiology [on line]. 2nd ed. New York: Springer, cop. 2012 [Consultation: 20/06/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3070377. ISBN 0387989021.
- Robinson, Clark. Dynamical systems : stability, symbolic dynamics, and chaos. 2nd ed. CRC Press, 1999. ISBN 0849384958.
- Meiss, J. D. Differential dynamical systems [on line]. Philadelphia: Society for Industrial & Applied Mathematics, 2007 [Consultation: 20/06/2023]. Available on: http://epubs.siam.org/doi/book/10.1137/1.9780898718232. ISBN 9780898716351.
- Strogatz, Steven H. Nonlinear dynamics and chaos : with applications to physics, biology, chemistry and engineering [on line]. Boulder: Westview Press, 2015 [Consultation: 23/06/2022]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1181622. ISBN 0429972199.
- Devaney, Robert L. An Introduction to chaotic dynamical systems. 2nd ed. Reading, MA [etc.]: Westview, 2003. ISBN 813340853.
- Sotomayor Tello, Jorge Manuel. Liçoes de equaçoes diferenciais ordinárias. Rio de Janeiro: Instituto de Matemática Pura e Aplicada (IMPA), 1979. ISBN 9216050624.
- Arrowsmith, D.K. ; Place, C.M. An Introduction to dynamical systems. Cambridge: Cambridge University Press, 1990. ISBN 0521303621.
- Guckenheimer, J. ; Holmes, P. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. New York, NY: Springer-Verlag, 1983. ISBN 0387908196.
- Pollard, Harry. Mathematical introduction to celestial mechanichs. Englewood Cliffs: Prentice-Hall, 1966.
- Meyer, Kenneth R.; Hall, G.R.; Offin, D. Introduction to Hamiltonian dynamical systems and the n-body problem [on line]. New York: Springer-Verlag, 2009 [Consultation: 20/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-09724-4. ISBN 9780387097237.
- Wiggins, Stephen. Introduction to applied nonlinear dynamical systems and chaos [on line]. 2nd ed. New York [etc.]: Springer-Verlag, cop. 2003 [Consultation: 20/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b97481. ISBN 9786610188161.