UNIVERSITAT POLITÈCNICA
DE CATALUNYA
BARCELONATECH

## Course guide

220001 - AL - Algebra

| Unit in charge: <br> Teaching unit: | Terrassa School of Industrial, Aerospace and Audiovisual Engineering <br>  <br> Degree: |
| :--- | :--- |
|  | BACHELOR'S DEGREE IN AEROSPACE TECHNOLOGY ENGINEERING (Syllabus 2010). (Compulsory subject). <br> BACHELOR'S DEGREE IN AEROSPACE VEHICLE ENGINEERING (Syllabus 2010). (Compulsory subject). |
| Academic year: 2023 | ECTS Credits: $6.0 \quad$ Languages: Catalan |

## LECTURER

## Coordinating lecturer:

## VICENÇ SALES I INGLÈS

## Others:

VICENÇ SALES i INGLÈS

## PRIOR SKILLS

Knowledges of secondary school mathematics.

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

## Specific:

CE01. The ability to solve mathematical problems that may arise in an engineering context. The ability to apply knowledge of linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation

## Transversal:

07 AAT N1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

## TEACHING METHODOLOGY

- The theory lessons would be intended to introduce the basic concepts of each topic, as well as examples and practical cases that allow each student to understand the topics. A basic reference will be a collection of slides that is freely available in the Atenea platform.
- The practical lessons would be intended to solve problems that are set before the lessons. The problems are also available freely in the Atenea platform. This book would help the students to get familiar with the basic concepts and the ability of expressing themselves properly.
- As a complement, a solved problem book will be also available.
- The teacher will set a fixed hour to solve doubts.
- There will be, along with the midterm and final exams, two evaluation tasks.


## LEARNING OBJECTIVES OF THE SUBJECT

- Comprehension and capability of application of the concept of linearity and its operative translation: matrix calculus.
- Operative part: calculus with matrices and determinants, study and resolution of linear equation systems, indistinct use of Gauss and minors methods and applicacions to conceptual part.
- Conceptual part: acquirement and assimilation of the basic notions of numerical vector and affine spaces (including derivated notions of inner product), linear and affine transformations (with special insistence in diagonalization and transformations with geometric meaning) and quadratic forms and varieties (specially with its study and classification).

STUDY LOAD

| Type | Hours | Percentage |
| :--- | :--- | :--- |
| Hours medium group | 28,0 | 18.67 |
| Self study | 90,0 | 60.00 |
| Hours large group | 32,0 | 21.33 |

Total learning time: 150 h

## CONTENTS

## I. MATRIX CALCULUS

## Description:

1. Matrices.
1.1. Matrices.

Operations. Transposition. Symmetric and antisymmetric matrices.
1.2. Elementary transformations below the pivot.

Rank and regular matrices. Elimination of linear parameters. Linear equations systems.
1.3. Elementary transformations below and above the pivot.

Simultaneous linear equations systems. Basic matrix equations of the product. Inverse matrix of a regular matrix.

## 2. Determinants.

2.1. Determinants.

Determinants. Direct, invers and singular matrices. Orthogonal matrices.
2.2. Minors.

Rank and regular matrices. Elimination of linear parameters. Linear equations systems.
2.3. Attached matrix.

Inverse matrix of a regular matrix. Basic matrix equations of the product. Simultaneous linear equations systems.

## Specific objectives:

Learn the necessary operational methodology for the conceptual part of the course.

## Related activities:

1, 2, 3, 5 and 6
Full-or-part-time: 37 h 30 m
Theory classes: 8h
Practical classes: 7 h
Self study : 22h 30m

## II. VECTOR AND AFFINE SPACES

## Description:

3. Vector spaces.
3.1. Vector space.

Addition and product by scalars. Linear dependence. Linear subspaces.
3.2. Inner product space.

Inner product and vectorial product. Orthogonal and orthonormal systems and bases. Orthogonal subspace.
3.3. Orientation and linear coordinates.

Positive and negative bases. Linear coordinates and base changes. Base orthogonal changes.
4. Affine spaces.
4.1. Affine space.

Addition of points and vectors and free vector. Linear varieties. Relative position of linear varieties.
4.2. Euclidean space.

Distance between points. Elementary volumes. Perpendicularity, angles and distances between linear varieties.
4.3. Orientation and affine coordinates.

Positive and negative references. Affine coordinates and reference changes. Reference orthogonal changes.

## Specific objectives:

Analyze concepts about points and vectors and the first degree relation between them.

## Related activities:

$1,2,5$ and 6
Full-or-part-time: 37h 30m
Theory classes: 8h
Practical classes: 7h
Self study : 22 h 30 m

## III. LINEAR AND AFFFINE TRANSFORMATIONS

## Description:

5. Linear transformations.
5.1. Linear transformations.

Matricial expression and base changes. Regular, singular, direct and inverse linear transformations. Diagonalization.
5.2. Linear transformations of inner product space.

Orthogonal diagonalization. Linear orthogonal projections and linear symmetries. Linear rotations.
5.3. Euclidean linear transformations.

Direct and inverse euclidean linear transformations. Linear compositions. Classification.
6. Related transformations.
6.1. Related transformations.

Matricial expression and reference changes. Regular, singular, direct and inverse related transformations. Fixed points.
6.2. Related transformations of euclidean space.

Related orthogonal projections and related orthogonal projections with sliding. Related symmetries and related symmetries with sliding. Related rotations and related rotations with sliding.
6.3. Euclidean related transformations.

Direct and inverse euclidean related transformations. Related compositions. Classification.

## Specific objectives:

Analyze the transformations of points and vectors and the relation between them.

## Related activities:

$1,2,4$ and 6
Full-or-part-time: 37h 30m
Theory classes: 8 h
Practical classes: 7h
Self study : 22h 30 m

## IV. QUADRATIC FORMS AND VARIETIES

## Description:

7. Quadratic forms and least squares.
7.1. Quadratic forms.

Matricial expression and base changes. Diagonalization. Linear classification.
7.2. Quadratic forms of inner product space.

Orthogonal base changes. Orthogonal diagonalization. Euclidean classification.
7.3. Least squares.

Linear interpretation of linear equation systems. Quadratic error. Least squares.
8. Quadratic varieties.
8.1. Quadratic varieties.

Matricial expression and reference changes. Degenerate quadratic varieties. Affine classification.
8.2. Quadratic varieties of euclidean space.

Reference orthogonal changes. Spheres. Euclidean classification.
8.3. Conics.

Ellipse. Hyperbola. Parable.
Specific objectives:
Analyze second-degree vector functions and second-degree relationships between points.

## Related activities:

1, 2 and 6
Full-or-part-time: 37 h 30 m
Theory classes: 8h
Practical classes: 7h
Self study : 22h 30m

## ACTIVITIES

## 1. THEORY SESSIONS

## Description:

Lectures.

## Specific objectives:

Theoretical concepts assimilation.

## Material:

See 'Bibligraphy' and 'Other resources'.
Full-or-part-time: 48h
Theory classes: 24h
Self study: 24 h

## 2. PRACTICAL SESSIONS

## Description:

Exercicses and problems classes.

## Specific objectives:

Resolution methods assimilation.

## Material:

See 'Bibliography' and 'Other resources'.
Full-or-part-time: 70h
Practical classes: 28h
Self study: 42h

## 3. FIRST CONTROL

## Description:

Exercises and problems of the chapters 1 and 2.

## Specific objectives:

Chapters 1 and 2 examination.

## Material:

See 'Bibliography' and 'Other resources'.

## Delivery:

Weight: 12,5\%.

## Full-or-part-time: 2h 30m

Theory classes: 1 h
Self study: 1h 30 m

## 4. SECOND CONTROL

## Description:

Exercises and problems of chapters 5 and 6.

## Specific objectives:

Chapters 5 and 6 examination.

## Material:

See 'Bibliography' and 'Other resources'.

## Delivery:

Weight: 12,5\%.
Full-or-part-time: 2h 30m
Theory classes: 1h
Self study: 1h 30 m

## 5. MIDTERM EXAM

## Description:

Exercicses and problems of chapters 1 to 4.

## Specific objectives:

Chapters 1 to 4 examination.

## Material:

See 'Bibliography' and 'Other resources'.

## Delivery:

Weight: 25\%.
Full-or-part-time: 12 h
Theory classes: 3h
Self study: 9 h

## 6. FINAL EXAM

## Description:

Exercises and problems of the chapters 1 to 8.

## Specific objectives:

All chapters examination.

## Material:

See 'Bibliography' and 'Other resources'.

## Delivery:

Weight: $50 \%$.
Full-or-part-time: 15 h
Theory classes: 3h
Self study: 12 h

## GRADING SYSTEM

$\mathrm{NF}=0.125 \mathrm{NC} 1+0.25 \mathrm{NEP}+0.125 \mathrm{NC} 2+0.50 \mathrm{NEF}$

NF: Global Qualification
NC1 : First Control Qualification
NEP: Midterm Exam Qualification
NC2 : Second Control Qualification
NEF : Final Exam Qualification

In case of failing the Partial Exam and passing the Final Exam, the mark of the Partial Exam will become 5.

## EXAMINATION RULES.

- During each of the assessment tests (exams and controls) each student must have an identification document (DNI, passport, UPC card, ...), which will be presented at the request of the teaching staff.
- In case of the lack of attendance to a particular evaluation task would lead to a grade of zero in that task. A valid justification, along with the document that evidences its veracity, would give the right to do the task in the later days.


## BIBLIOGRAPHY

## Complementary:

- Amer Ramon, R; Carreras Escobar, F.. Curs d'Àlgebra Lineal. 2a ed. Terrassa: Universitat Politècnica de Catalunya, 1998. ISBN 8484987841.


## RESOURCES

## Other resources

- Àlgebra Lineal. Transparències (available in ATENEA)
- Àlgebra Lineal. Exercicis resolts (available in ATENEA)
- Àlgebra Lineal. Exercicis (available in ATENEA)

