

## Course guide

# 220125 - NS - Nonlinear Systems, Chaos and Control in Engineering

**Last modified:** 11/04/2025

**Unit in charge:** Terrassa School of Industrial, Aerospace and Audiovisual Engineering  
**Teaching unit:** 748 - FIS - Department of Physics.

**Degree:** BACHELOR'S DEGREE IN AEROSPACE TECHNOLOGY ENGINEERING (Syllabus 2010). (Optional subject).  
BACHELOR'S DEGREE IN AEROSPACE VEHICLE ENGINEERING (Syllabus 2010). (Optional subject).  
BACHELOR'S DEGREE IN INDUSTRIAL TECHNOLOGY ENGINEERING (Syllabus 2010). (Optional subject).

**Academic year:** 2025    **ECTS Credits:** 3.0    **Languages:** English

### LECTURER

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**Coordinating lecturer:** Cristina Masoller (cristina.masoller@upc.edu)

**Others:** Antonio Pons (a.pons@upc.edu)

### REQUIREMENTS

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The course requires a good knowledge of matlab or any similar computer program that allows to perform numerical simulations and to graphically visualize the results.

The students are expected to attend the lectures with their own portable computer with matlab or similar. If a student does not have a portable computer with matlab or similar, to attend the lectures he or she should borrow a portable computer with matlab from UPC library in Terrassa.

### TEACHING METHODOLOGY

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**Theory classes:** The course is divided into three parts, that will introduce the theory of nonlinear systems in a systematic way, starting with a general overview, then presenting analytical methods for one-dimensional systems (fixed points and their bifurcations), followed by two-dimensional systems (limit cycles and their bifurcations) and culminating with three dimensional systems, chaos, fractals and strange attractors.

Mathematical concepts will be gradually introduced. Emphasis will be given to specific examples (e.g. mechanical vibrations, lasers, chaotic circuits) that will facilitate the understanding of the theory of nonlinear systems and emphasize its relevance for technological applications.

**Practical classes:** hands-on sessions to solve problems via computer simulations will be offered.

**Self-study for doing exercises and activities:** The students will work in small groups (2-3 students) the problems proposed by the Lecturers.

The lecturers will provide the required materials and will monitor the activities via ATENEA.

### LEARNING OBJECTIVES OF THE SUBJECT

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Acquire a general understanding of the behavior of nonlinear systems and chaotic systems, with emphasis on control techniques and their practical applications to Engineering problems.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	30,0	40.00
Self study	45,0	60.00

**Total learning time:** 75 h

## CONTENTS

### Module 1: Introduction and analysis of one-dimensional nonlinear systems

#### Description:

There will be a broad overview of nonlinear and chaotic systems and several examples will be discussed. The relevance of nonlinear effects and dynamical effects will be stressed in relation with linear approximations.

#### Specific objectives:

1. One dimensional systems will be analyzed, employing linear stability analysis and going beyond to the concept of bifurcations.
2. Control of nonlinear systems through delay feedback will be studied.
3. Practical examples to be discussed include a laser model, a neuron model, an ecological system and an over-damped pendulum.

#### Related activities:

The students will be offered hands-on sessions on computer simulations on integrating the model equations and computing bifurcation diagrams.

**Full-or-part-time:** 25h

Theory classes: 10h

Self study : 15h

### Module 2: Two-dimensional Flows

#### Description:

1. Two-dimensional linear systems will be analyzed in detail. As a working example, the harmonic oscillator will be considered to demonstrate different concepts as vector fields on the phase space or phase portraits.
2. The dynamics of the phase space will be characterized for two-dimensional nonlinear systems. Fixed points and linearization will be presented. Dynamics of different systems coming from Physics and Technology will be worked out as examples of the techniques presented in this block.
3. Limit cycles of two-dimensional flows will be studied paying special attention to their Bifurcation analysis. Hysteresis, coupled oscillators and maps will be introduced.

#### Related activities:

Hands-on group sessions on numerical techniques and simulations for two-dimensional dynamical systems. Computer code for these sessions will be programmed in Matlab.

**Full-or-part-time:** 25h

Theory classes: 10h

Self study : 15h



### Module 3: Chaotic dynamics

**Description:**

1. In this module we will discuss examples of chaotic nonlinear systems in three dimensions. Examples will include the Lorenz system and its technological applications, and chaotic electronic circuits.
2. The last part of the course will be devoted to characterization of chaos. We will discuss the exponential separation of trajectories and the concept of Lyapunov exponents. Finally, we will discuss chaotic attractors and introduce fractals and the idea of fractal dimension.

**Related activities:**

Hands-on session on numerical simulation of chaotic systems. Numerical estimation of Lyapunov exponents and fractal dimension with Matlab.

**Full-or-part-time:** 25h

Theory classes: 10h

Self study : 15h

## GRADING SYSTEM

The students will have to present three reports, one for each module of the course.

Two of the reports will be written while the third one will be a short oral presentation (5-10 minutes depending on the number of students) that will be followed by questions.

The final grade will be the average of the three grades obtained in the three reports:

$$NF = 0.333 \cdot N1 + 0.333 \cdot N2 + 0.333 \cdot N3$$

Where NF is the final grade and N1, N2, N3 are the grades obtained in three reports.

If any student wants to improve the final grade, he or she will be given the opportunity of a second oral presentation, within the next 10 days of the first oral presentation.

The students that are unsatisfied with they final grades will be given the opportunity to re-do the report that had the lowest grade.

## EXAMINATION RULES.

The students will present three reports, one for each module of the course. Two of the reports will be written while the third one will be a short oral presentation (5-10 minutes depending on the number of students) that will be followed by questions. The instructors will decide which report will be presented orally.

The grades obtained in the reports will take into account attendance and active participation in class. Reports received up to 48 hours after the deadline will be penalized by 50% and will not be accepted after that.

If any student wants to improve the final grade, he or she will be given the opportunity of a second oral presentation, within the next 10 days of the first oral presentation.

## BIBLIOGRAPHY

**Basic:**

- Strogatz, Steven H. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry and engineering [on line]. 2nd ed. Philadelphia: Westview Press, 2015 [Consultation: 04/07/2025]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1181622>. ISBN 9780813349107.

**Complementary:**



- Jost, Jürgen. Dynamical systems: examples of complex behaviour [on line]. Berlin, Heidelberg: Springer-Verlag, 2005 [Consultation: 15/06/2022]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/3-540-28889-9>. ISBN 9783540288893.
- Wiggins, Stephen. Introduction to applied nonlinear dynamical systems and chaos [on line]. 2nd ed. New York [etc.]: Springer-Verlag, 2003 [Consultation: 20/09/2022]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/b97481>. ISBN 0387001778.
- Ott, Edward. Chaos in dynamical systems. 2nd. ed. Cambridge: University Press, 2002. ISBN 0521010845.
- Pikovsky, A.; Rosenblum, M.; Kurths, J. Synchronization: a universal concept in nonlinear sciences. Cambridge: Cambridge University Press, 2001. ISBN 9780521533522.

## RESOURCES

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### Hyperlink:

- TED Lectures. [http://www.ted.com/talks/steven\\_strogatz\\_on\\_sync](http://www.ted.com/talks/steven_strogatz_on_sync)