

## Course guide

### 230080 - AL - Linear Algebra

**Last modified:** 25/05/2023

**Unit in charge:** Barcelona School of Telecommunications Engineering  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** BACHELOR'S DEGREE IN TELECOMMUNICATIONS TECHNOLOGIES AND SERVICES ENGINEERING (Syllabus 2015). (Compulsory subject).

**Academic year:** 2023    **ECTS Credits:** 6.0    **Languages:** Catalan, Spanish

#### LECTURER

---

**Coordinating lecturer:** Consultar aquí / See here:  
<https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/responsables-assignatura>

**Others:** Consultar aquí / See here:  
<https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/professorat-assignat-idioma>

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

---

**Generical:**

2. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

**Transversal:**

1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

#### TEACHING METHODOLOGY

---

Application lectures  
Expositive lectures  
Personal work (non classroom)  
Short-answer questions (Test)  
Proves de resposta llarga (Examen Final)

#### LEARNING OBJECTIVES OF THE SUBJECT

---

To introduce the basic concepts of linear algebra.

Learning outcome:

He/she expresses clearly the process of planning and solving exercises and problems that require the use of linear algebra.  
He/she understands and masters the most useful methods to solve problems in the area of this subject.  
He/she addresses numerical description and formulation of problems with descriptive description.  
He/she makes use of more than one source and uses it in a complementary manner to observe the events described in the main text.  
He/she identifies problems and models from open situations and explores alternative resolutions.

## STUDY LOAD

Type	Hours	Percentage
Self study	85,0	56.67
Hours large group	65,0	43.33

**Total learning time:** 150 h

## CONTENTS

### (ENG) Tema 1. Matrices and determinants.

#### Description:

Matrices and sub-matrices. Operations and properties.  
Elementary transformations. Echelon forms.  
Rank of a matrix. Inverse matrix.  
Systems of linear equations. Discussion and resolution of systems.  
Gaussian elimination. Gauss-Jordan elimination.  
Determinants: definition and properties.  
Calculation of determinants. Orthogonal matrices.  
Minors and calculating the rank of a matrix by minors. Cramer's rule.  
Traces and cofactors. Laplace's formula. Adjugate matrix.

#### Related competencies :

08 CRPE N1. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

07 AAT N1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

#### Full-or-part-time: 23h

Theory classes: 10h

Self study : 13h

### (ENG) Tema 2. Vectorial spaces.

**Description:**

Definition, properties, and examples.

Linear independence.

Generating system, basis and dimension.

Components and change of basis.

Vector subspaces. Implicit equations.

Intersection, sum and direct sum. Grassmann formula.

The four subspaces associated to a matrix.

**Related competencies :**

08 CRPE N1. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

07 AAT N1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

**Full-or-part-time:** 28h 45m

Theory classes: 12h 30m

Self study : 16h 15m

### (ENG) Tema 3. Euclidean space.

**Description:**

Inner product, norm, and angle.

Cauchy-Schwarz and triangular inequalities, Pythagorean theorem.

Orthogonality. Orthonormal and orthonormal basis.

Change of basis. Positive definite matrices.

Orthogonal complement. Orthogonal projection and best approximation.

Gram-Schmidt method. Normal equations and Fourier coefficients.

Best approximation for a linear system: least squares. Quadratic error.

Orthogonality of the fundamental subspaces.

Euclidean vector spaces of infinite dimension. Orthogonal polynomials and trigonometric functions.

Introduction to unitary space.

**Related competencies :**

08 CRPE N1. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

07 AAT N1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

**Full-or-part-time:** 34h 30m

Theory classes: 15h

Self study : 19h 30m

#### (ENG) Tema 4. Linear transformations.

##### Description:

Definition and properties. Associated matrix.  
Image and inverse image. Change of basis. Similar matrices.  
Rank of a linear transformation.  
Kernel and image. Rank-nullity theorem.  
Injective and exhaustive transformations.  
Endomorphisms. Change of basis. Equivalent matrices.  
Invariant subspaces.

##### Related competencies :

08 CRPE N1. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

07 AAT N1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

##### Full-or-part-time: 23h

Theory classes: 10h

Self study : 13h

#### (ENG) Tema 5. Diagonalization of endomorphisms and singular value decomposition

##### Description:

Eigenvectors and eigenvalues.  
Characteristic polynomial and traces of an endomorphism.  
Eigenspaces, algebraic and geometric multiplicities.  
Diagonalization conditions.  
Complex eigenvalues of real matrices.  
Symmetric endomorphisms.  
Orthogonal basis of eigenvectors.  
Orthogonal diagonalization of symmetric matrices. Spectral theorem.  
Positive definite and semidefinite matrices.  
Singular value decomposition.

##### Related competencies :

08 CRPE N1. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

07 AAT N1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

##### Full-or-part-time: 28h 45m

Theory classes: 12h 30m

Self study : 16h 15m

## ACTIVITIES

---

### (ENG) Test (Test)

**Full-or-part-time:** 1h

Theory classes: 1h

### (ENG) Test (Test)

**Full-or-part-time:** 1h

Theory classes: 1h

### (ENG) Exam (Final Exam)

**Full-or-part-time:** 3h

Theory classes: 3h

## GRADING SYSTEM

---

Two tests along course: 40%

Final exam: 60%

## BIBLIOGRAPHY

---

### Basic:

- Amer, R.; Carreras, F. Curs d'àlgebra lineal. 2a ed. Terrassa: Universitat Politècnica de Catalunya, 1998. ISBN 8484987841.
- Strang, G. Introduction to linear algebra. 6th ed. Wellesley, Mass.: Wellesley-Cambridge Press, 2023. ISBN 9781733146678.
- Lay, D.C. Álgebra lineal y sus aplicaciones [on line]. 5a. ed. Madrid: Pearson Educación, 2016 [Consultation: 18/05/2020]. Available on: [http://www.ingebook.com/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=6765](http://www.ingebook.com/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=6765). ISBN 9786073237451.

### Complementary:

- Rojo García, J.; Martín, I. Ejercicios y problemas de álgebra lineal. 2a ed. Madrid: McGraw-Hill, 2004. ISBN 8448198581.
- Amer, R; Sales, V. Àlgebra lineal: problemes resolts [on line]. Barcelona: Universitat Politècnica de Catalunya, 2009 [Consultation: 10/10/2022]. Available on: <http://mat-web.upc.edu/people/rafael.cubarsi/algebra/algebra-lineal-problemes-resolts.pdf>. ISBN 8476532768.