

## Course guide

### 230489 - RELG - General Relativity

Last modified: 09/11/2022

**Unit in charge:** Barcelona School of Telecommunications Engineering  
**Teaching unit:** 749 - MAT - Department of Mathematics.  
748 - FIS - Department of Physics.

**Degree:** BACHELOR'S DEGREE IN ENGINEERING PHYSICS (Syllabus 2011). (Optional subject).

**Academic year:** 2022    **ECTS Credits:** 6.0    **Languages:** Spanish

#### LECTURER

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**Coordinating lecturer:** Consultar aquí / See here:  
<https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/responsables-assignatura>

**Others:** Consultar aquí / See here:  
<https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/professorat-assignat-idioma>

#### PRIOR SKILLS

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Knowledge on classical Mechanics and classical Electromagnetism acquired in the previous subjects of the Degree; in particular, it is advisable to have studied the subject "Mechanics" (2A, Degree on Physics Engineering), or "Mathematical Models of Physics" (3B, Degree on Mathematics).

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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##### Specific:

MAT1. Ability to solve math problems that may arise in engineering. Ability to apply knowledge about linear algebra, geometry, differential geometry, differential and integral calculus, ordinary and partial differential equations, probability and statistics.

3. Knowledge of the scientific method and its applications in physics and engineering. Ability to formulate hypotheses and make critical analysis of scientific problems in the field of physics and engineering. Ability to relate the physical reality with their mathematical models and vice versa.

FG2. Ability to solve basic problems in mechanics, elasticity, thermodynamics, fluids, waves, electromagnetism and modern physics, and its application in solving engineering problems.

##### Generical:

2. ABILITY TO IDENTIFY, FORMULATE, AND SOLVE PHYSICAL ENGINEERING PROBLEMS. Planning and solving physical engineering problems with initiative, making decisions and with creativity. Developing methods of analysis and problem solving in a systematic and creative way.

##### Transversal:

1. SELF-DIRECTED LEARNING - Level 3. Applying the knowledge gained in completing a task according to its relevance and importance. Deciding how to carry out a task, the amount of time to be devoted to it and the most suitable information sources.

#### TEACHING METHODOLOGY

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Lectures with interaction with students

## LEARNING OBJECTIVES OF THE SUBJECT

The aims of this course are to provide sufficient training to students in order to advance the knowledge of the Theory of General Relativity (TRG). After introducing the basic mathematical tools and reviewing the Minkowskian formulation of the Theory of Special Relativity, we want to describe the conceptual progress that led Albert Einstein to formulate the TRG and we state this theory, its current relevance, the experimental tests that support it and the main contributions in the subject of the external field and stellar collapse, generation of black holes and the origin and evolution of the universe: cosmological solutions.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	65,0	43.33
Self study	85,0	56.67

**Total learning time:** 150 h

## CONTENTS

### 1. Complements of tensor algebra and differential geometry.

#### Description:

Basic concepts on tensor algebra. Tensor fields. Operations. Vector fields, differential forms and tensor fields in  $\mathbb{R}^n$ . Differentiable manifolds. Differentiable operators. Derivations. Covariant derivative. Christoffel symbols. Connections. Parallel transport. Geodesics. Equations. Metric tensor. Symmetries and Killing vectors. Torsion tensor of a connection. Levi-Civita connection. Curvature tensors (Riemann, Ricci), properties.

**Full-or-part-time:** 47h 20m

Theory classes: 10h

Practical classes: 10h

Self study : 27h 20m

### 2. Review on Special Relativity: The Minkowskian formulation of the Special Relativity.

#### Description:

Postulates of the Special Relativity. Minkowski's metrics and Minkowskian space-time. Inertial observers. Four-vectors. Light cone. Lorentzian geometry. Lorentz and Poincaré transformations and groups. Relativistic kinematics and relativistic dynamics. The electromagnetic tensor: Maxwell equations.

Four-interval. Light cone and causality. Four-velocity and four-acceleration: transformation laws. Equations of relativistic dynamics: Conservation of four-momentum and relativistic Newton's law. Invariants and Minkowski metric. The space-time of the Special Relativity. The electromagnetic tensor: Maxwell equations.

**Full-or-part-time:** 7h

Theory classes: 2h

Practical classes: 1h

Self study : 4h

### 3. Principles of General Relativity.

**Description:**

Foundations of General Relativity, Newton's equations of gravitation. Inertial and gravitational mass. The principle of equivalence, consequences. Non-Euclidean geometries. Postulates of General Relativity.

**Full-or-part-time:** 3h 30m

Theory classes: 1h

Practical classes: 1h

Self study : 1h 30m

### 4. The equations of General Relativity.

**Description:**

The stress-energy-momentum tensor. Conservation of the stress-energy-momentum tensor. The Einstein tensor. Properties. Einstein field equations and geodesic equation. Consequences of Einstein equations. Tidal effects. The cosmological constant. General relativity from a variational principle: the Hilbert-Einstein Lagrangian. Phenomenological aspects.

**Full-or-part-time:** 11h 50m

Theory classes: 3h 20m

Practical classes: 1h 40m

Self study : 6h 50m

### 5. Kinematics in General Relativity.

**Description:**

Time-like geodesic congruencies. Distances and time intervals in general relativity. Three-dimensional metric tensor. The synchronization criteria of clocks in General Relativity. Locally-inertial reference frame. Gravitational red-shift. A covariant generalization of Doppler effect and gravitational red-shift.

**Full-or-part-time:** 6h 30m

Theory classes: 2h

Practical classes: 2h

Self study : 2h 30m

### 6. The Schwarzschild solution. Geodesics in Schwarzschild geometry.

**Description:**

Solutions with spherical symmetry. Static solutions. Asymptotically flat solutions. The Schwarzschild spacetime. Properties and the Birkhoff theorem. Singularities of the curvature (essential, intrinsic or real) and removable singularities (singularities of the coordinates). Lagrangian method to obtain the equation of timelike (or null) geodesics. Symmetries and conserved quantities. Bound orbits. Stable and unstable circular orbits. Radial geodesics.

**Full-or-part-time:** 16h 20m

Theory classes: 4h

Practical classes: 3h

Self study : 9h 20m

## 7. Experimental tests of General Relativity.

### Description:

Advance of the perihelion of Mercury. Deflection of light rays. Gravitational Red-Shift. The electromagnetic waves time delay.

**Full-or-part-time:** 7h 10m

Theory classes: 2h

Practical classes: 1h

Self study : 4h 10m

## 8. Black holes.

### Description:

Eddington-Finkelstein coordinates. Event horizon. Black holes. A classical argument. Tidal forces in a black hole. Observational evidence for black holes.

**Full-or-part-time:** 11h 50m

Theory classes: 3h

Practical classes: 2h

Self study : 6h 50m

## 9. Maximal extension and conformal compactification.

### Description:

Maximal analytic extensions. The Kruskal solution. Penrose diagram of Minkowski's space-time. Penrose diagram of the Schwarzschild-Kruskal space-time. Black holes versus white holes. Closed trapped surfaces. Gravitational collapse of a star. Creation of black holes.

**Full-or-part-time:** 11h 40m

Theory classes: 3h

Practical classes: 2h

Self study : 6h 40m

## 10. Radiation modeling and collapse: Vaidya solution

### Description:

External spacetime of a star that emits or receives radiation: Vaidya metric. Flux of radiation. Stress-energy-momentum tensor. Black holes and radiation. Penrose diagrams of Vaidya metric. Radiating collapse of a spherically symmetric space-time: Matching conditions. Dominant energy conditions in a radiative collapse.

**Full-or-part-time:** 11h

Theory classes: 2h

Practical classes: 2h

Self study : 7h

## 11. Relativistic Cosmology. Cosmological models.

### Description:

Olbers' Paradox. Hubble's Law. The cosmological principle. Weyl's postulate. Friedmann equations. Relativistic cosmology. The geometry of 3-spaces of constant curvature. The flat space models. Friedmann-Lemaître-Robertson-Walker models in a flat case. Big-Bang. Conformal structure of a "flat" Friedmann-Lemaître-Robertson-Walker model.

**Full-or-part-time:** 15h 50m

Theory classes: 4h

Practical classes: 3h

Self study : 8h 50m

## GRADING SYSTEM

Two partial exams (P1 and P2). Final exam (F) (optional).

Final qualification:  $(P1+P2)/2$  or F.

Extraordinary exam (E): it will be carried out only in the case that the subject is not approved in the ordinary call and will include all the contents of the course. The final qualification of the subject will be E.

## BIBLIOGRAPHY

### Basic:

- D'Inverno, R. Introducing Einstein's relativity. Oxford: Clarendon Press . Oxford University Press, 1992. ISBN 0198596863.
- Misner, C.W.; Thorne, K.S.; Wheeler, J.A. Gravitation. San Francisco: W. H. Freeman and Company, 1973. ISBN 0716703440.
- Landau, L.D. The classical theory of fields [on line]. 4th rev English ed. Oxford: Pergamon, 1980 [Consultation: 10/10/2018]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?docID=1675199>. ISBN 9781483293288.
- Carroll, S.M. Spacetime and geometry: an introduction to general relativity. New Intern. ed. Essex: Pearson, 2014. ISBN 9781292026633.
- Poisson, E. A relativist's toolkit: the mathematics of black-hole mechanics. Cambridge, UK ; New York: Cambridge University Press, 2004. ISBN 0521830915, 9780511606601.