



## Course guide

### 230903 - C - Calculus

Last modified: 25/05/2023

**Unit in charge:** Barcelona School of Telecommunications Engineering  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** BACHELOR'S DEGREE IN ELECTRONIC ENGINEERING AND TELECOMMUNICATION (Syllabus 2018).  
(Compulsory subject).

**Academic year:** 2023    **ECTS Credits:** 6.0    **Languages:** Catalan

#### LECTURER

**Coordinating lecturer:** Consultar aquí / See here:  
<https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/responsables-assignatura>

**Others:** Consultar aquí / See here:  
<https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/professorat-assignat-idioma>

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

##### Specific:

CE3. (ENG) GREELC: Comprensió i domibi dels conceptes bàsics sobre les lleis generals de la macànica, termodinàmica, camps i ones i electromagnetisme i la seva aplicació per a la resolució de problemes propis de l'enginyeria. (Mòdul de formació bàsica).

##### Generalic:

CG3. (ENG) GREELEC: Coneixmetn de matèries bàsiques i tecnològiques que el capacitin per a l'aprenentatge de nous mètodes i tecnologies, així com que el dotin d'una gran versatilitat per adaptar-se a noves situacions.

##### Transversal:

CT6. (ENG) GREELEC: APRENENTATGE AUTÒNOM: Detectar deficiències en el propi coneixement i superarles mitjançant la reflexió crítica i l'elecció de la millor actuació per ampliar coneixements.

##### Basic:

CB1. (ENG) GREELEC: Que els estudiants hagin demostrat tenir i comprendre coneixements en una àrea d'estudi que neix de la base de l'educació secundària general, i que sol trobar un nivell que, si bé es recolza en llibres de text avançats, inlou també alguns aspectes que impliquin coneixements procedents de la vanguardia del seu camp d'estudi.

#### TEACHING METHODOLOGY

Problem solving classes

On campus lessons

Individual work (no face to face lessons)

Short answer controls and homework

Final exam (long answer exam)



## LEARNING OBJECTIVES OF THE SUBJECT

Achieving sufficient level of one variable Calculus to deal with, or to base the treatment of phenomena that can be described in these terms. Also support of parties other subjects that require mastery of real functions of one variable. Introduction to functions defined by series.

Learning outcomes:

Clearly expresses the process of planning and problem solving, and problems that require the use of calculus of one variable.  
Comprehend and dominates the most useful methods for solving problems in the field of one variable.  
He/she is able to confront the equations and numerical description of problems with descriptive statement.  
He/she uses more than one source, and uses it as complementary to observe the events described in the main text.  
Identifies problems and models from open situations. Study alternatives for their resolution.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	65,0	43.33
Self study	85,0	56.67

**Total learning time:** 150 h

## CONTENTS

### Unit 1. Real numbers

#### Description:

Numerical sets (natural, integers and rational). The field of the real numbers. Order relation, inequalities. Absolute value. Maximum, minimum, supremum and infimum of subsets of R. Intervals.

#### Full-or-part-time: 11h 30m

Theory classes: 5h

Self study : 6h 30m

### Unit 2. Functions

#### Description:

Real function of one real variable. Domain and range. Graphic of a function. Injective, exhaustive and bijective functions. Inverse function. Operations between functions. Elementary functions: polynomials, rational, potential, exponential, logarithm, trigonometric and hyperbolic.

#### Full-or-part-time: 16h 06m

Theory classes: 7h

Self study : 9h 06m



### Unit 3. Limits of functions

**Description:**

Limit of a function at a point . Infinite limits . Limits at infinity. Properties of the limit, algebra of infinite limits. Lateral limits . Uncertainties : infinite/infinite (rational functions), infinity-infinity (difference of roots or logarithms),  $1^{\infty}$ infinity (number e).

**Full-or-part-time:** 16h 06m

Theory classes: 7h

Self study : 9h 06m

### Unit 4. Continuity

**Description:**

Continuous functions: definition and properties. Types of discontinuity. Bounds, maximum and minimum of functions. Weierstrass theorem. Bolzano theorem. Mean value theorem.

**Full-or-part-time:** 9h 21m

Theory classes: 4h

Self study : 5h 21m

### Unit 5. Differentiability

**Description:**

Derivative of a function at a point, derivative function. Tangent line. Properties of the derivative (Leibnitz, chain rule, inverse function). Derivative of elementary functions. Rolle theorem. Mean value theorems. L'Hôpital theorem and application to computation of limits. Infinitesimals and infinites.

**Full-or-part-time:** 16h 06m

Theory classes: 7h

Self study : 9h 06m

### Unit 6. Taylor polynomials

**Description:**

Contact order. Taylor polynomials of a function. Taylor residue. Taylor formula. Taylor polynomials of elementary functions. Properties of the Taylor polynomials. Applications: approximation, computation of limits.

**Full-or-part-time:** 11h 30m

Theory classes: 5h

Self study : 6h 30m

### Unit 7. Analysis of the variation of functions

**Description:**

Increasing and decreasing functions. Local extrema. Concavity and convexity. Inflection points. Characterization from the signs of the derivatives. Asymptotes. Graphical study of functions.

**Full-or-part-time:** 9h 12m

Theory classes: 4h

Self study : 5h 12m



## Unit 8. Primitives

**Description:**

Primitive of a function. Calculation of primitives: immediate, by parts and by change of variable. Calculation of primitive of rational, trigonometric and irrational functions.

**Full-or-part-time:** 16h 06m

Theory classes: 7h

Self study : 9h 06m

## Unit 9. Riemann's Integral

**Description:**

Definition of the Riemann's integral. Properties. Fundamental theorem of calculus. Applications of the integral.

**Full-or-part-time:** 9h 12m

Theory classes: 4h

Self study : 5h 12m

## Unit 10. Improper integrals.

**Description:**

Locally integrable functions. Improper integrals of the first kind. Improper integrals of the second kind. Convergence criteria. Absolute convergence. Euler gamma function.

**Full-or-part-time:** 9h 12m

Theory classes: 4h

Self study : 5h 12m

## Unit 11. Numerical series and power series

**Description:**

Sequences. Numerical series. Examples (geometric and harmonic series). Convergence criteria (comparison, root, quotient, integral). Alternated series. Absolute convergence. Power series. Radius and interval of convergence. Derivation and integration of functions defined by power series. Taylor series.

**Full-or-part-time:** 13h 48m

Theory classes: 6h

Self study : 7h 48m

## Unit 12. Numerical methods

**Description:**

Solution of equations: bisection method, Newton method, iteracions and fixed points. Interpolation of functions: Lagrange polynomials. Numerical derivatives and integrals.

**Full-or-part-time:** 11h 30m

Theory classes: 5h

Self study : 6h 30m



## ACTIVITIES

### FINAL EXAMEN

**Description:**

Final exam

**Full-or-part-time:** 3h

Theory classes: 3h

### CONTROL

**Description:**

Short answer controls

**Full-or-part-time:** 3h

Theory classes: 3h

## GRADING SYSTEM

Kind of exams to do and weight on the final evaluation:

Final exam: 60%

Continuous evaluation: 40%

On this subject will be evaluated the degree competences:

-Self-directed learning (Elementary level)

-Ability to identify, formulate and solve engineering problems (Elementary level)

## EXAMINATION RULES.

The standard ones for this kind of controls

## BIBLIOGRAPHY

**Basic:**

- Gracia, I.; Padró, C. Apunts de teoria per a l'assignatura de càlcul. (Atenea) [on line]. [Consultation: 13/05/2020]. Available on: <https://atenea.upc.edu/login/index.php>.

- Aguiló, F. [et al.]. Aprendentatge de càlcul [on line]. Barcelona: Edicions UPC, 2002 [Consultation: 04/03/2015]. Available on: <http://hdl.handle.net/2099.3/36227>. ISBN 8483016311.

- Aroca, Josep Maria. Càlcul infinitesimal: notes de classe [on line]. Barcelona: Departament de Matemàtica Aplicada IV, Universitat Politècnica de Catalunya, 2014 [Consultation: 10/10/2022]. Available on: <https://web.mat.upc.edu/josep.m.aroca/calcul/calcul-apunts-jmaroca.pdf>.

- Spivak, M. Calculus [on line]. 3a. ed. Barcelona: Reverte, 2012 [Consultation: 04/05/2020]. Available on: [http://www.ingebook.com/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=8020](http://www.ingebook.com/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=8020). ISBN 9788429151824.

**Complementary:**

- Professors de l'assignatura. Col·lecció d'exercicis de càlcul. (Atenea) [on line]. 2009 [Consultation: 14/05/2020]. Available on: <https://atenea.upc.edu/login/index.php>.

- Baranenkov, G.; Demidovich, B. P. Problemas y ejercicios de análisis matemático. Madrid: Paraninfo, 1969. ISBN 8428300496.

- Spivak, M. Answer book for calculus. 3rd. ed. Publish or Perish, ISBN 9780914098904.