

Course guide

2500001 - GECALGELIN - Linear Algebra

Last modified: 01/10/2023

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2020). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan, Spanish, English

LECTURER

Coordinating lecturer: NAPOLEON ANENTO MORENO

Others: NAPOLEON ANENTO MORENO, FRANCISCO JAVIER MARCOTE ORDAX, FRANCISCO JAVIER OZON GORRIZ

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
14392. Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge about: linear algebra; geometry; differential geometry; differential and integral calculation; differential equations and partial derivatives; numerical methods; numerical algorithmic; Statistics and optimization. (Basic training module)

TEACHING METHODOLOGY

The course consists of 4 hours per week of classroom activity (large size group) and optionally 2 hours weekly for a workshop (medium size group).

The 2 hours in the large size groups are devoted to theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises.

Another 2 hours in the large size groups is devoted to solving practical problems with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

In the 2 hours of workshop (medium size group), the students receive academic support to facilitate the understanding of the course. This support is materialized in two ways: (1) reminder of basic mathematical concepts which are essential to learn the new concepts introduced in the course; (2) guided problem solving of additional problems and tests from previous years.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENEA: content, program of learning and assessment activities conducted and literature.

Although most of the sessions will be given in the language indicated, sessions supported by other occasional guest experts may be held in other languages.

LEARNING OBJECTIVES OF THE SUBJECT

Knowledge of vector spaces; Matrices; Determinants; Linear equation systems; Linear applications; Euclidean spaces; Reduction of endomorphisms and matrices; and symmetric and orthogonal operators.

- 1 Ability to interpret vector spaces.
- 2 Ability to solve linear equations systems both manually and through some basic computer program.
- 3 Ability to produce geometric interpretations of concepts in vector calculus.

Knowledge of linear algebra, methods of solving linear problems that appear in engineering, elements of analytical geometry. Capacity for solving the mathematical problems posed in engineering involving these concepts. Knowledge of vector spaces. Knowledge of systems of linear equations, basic algorithms for the solution. Knowledge of analytical geometry. Knowledge of linear operators: endomorphisms and spectral theorems, related Euclidean spaces, eigenvalues and eigenvectors. Knowledge of determinants and their applications, particularly in the calculation of areas and volumes.

STUDY LOAD

Type	Hours	Percentage
Self study	84,0	56.00
Guided activities	6,0	4.00
Hours medium group	30,0	20.00
Hours large group	30,0	20.00

Total learning time: 150 h

CONTENTS

1. Vector space

Description:

Definition and examples. Vector subspaces. Linear dependence and independence.
 Generator system. Spaces of finite dimension. Bases.
 Range of a vector system. Intersection and sum of subspaces. Direct sum. Grassmann
 Basic problems
 Solving problems of dependence and linear independence. Calculation of bases. Component calculation.
 Problems of intersection and sum of subspaces.

Specific objectives:

Vector spaces constitute the general framework of applications in engineering and not just three-dimensional space.

Full-or-part-time: 26h 24m

Theory classes: 6h

Practical classes: 5h

Self study : 15h 24m

2. Matrices and systems of linear equations

Description:

Definitions and types. Sum of matrices. Scalar product- Matrix product. Elementary row and column operations. Reduced row echelon form. Regular matrices: calculation of the inverse by the Gauss-Jordan method

Basic problem solving

Systems of linear equations. Equivalent systems. Rouché-Fröbenius theorem. Gauss-Jordan reduction.

Solving systems of linear equations. Applications

Specific objectives:

Matrices are the fundamental tools we will work with and will then use in engineering applications. It is necessary to know in detail its basic properties.

From the properties of the matrices and especially using the elementary row operations the solution of the systems of linear equations is considered. Numerical resolution algorithms are introduced.

Full-or-part-time: 24h

Theory classes: 5h

Practical classes: 5h

Self study : 14h

3. Determinant

Description:

Definition of a determinant. Fundamental properties. Determinant of a triangular matrix. Determinant of a block diagonal matrix. Calculation of determinants. Gauss method.

Expression of the determinant. Development of a row and a column. Determinant of matrix multiplication. Determinants and matrix inversion. Cramer's rule. Geometric applications: Volume of a parallelepiped. Vector product. Properties.

Examples of determinants calculated by reducing the matrix to triangular form.

Exercises to determine if a matrix is invertible, and if so get its inverse. Solving systems of linear equations using Cramer's Rule.

Specific objectives:

Define the alternating multilinear forms, of which the determinant is a special case. From its definition shows some basic properties, all without having to explain the development of the determinant. Calculate the determinant of a matrix by applying numerical row elementary operations to reduce it to the triangular shape.

After introducing the basic properties of permutations make explicit the crucial and its development. Using the properties of alternating multilinear forms of showing that the determinant of a matrix is the product of determinants. Cofactor matrix is defined and used in the calculation of the inverse matrix.

Practicing proper elementary row operations and become aware of the possible numerical programming method.

Working this second method to invert a matrix, as already know to transform it into reduced row echelon form by row.

Full-or-part-time: 19h 12m

Theory classes: 4h

Practical classes: 4h

Self study : 11h 12m

4. Linear maps

Description:

Definitions and examples. Image and Kernel subspaces. Monomorphism, epimorphism, isomorphism. Basic properties. Vector space of linear maps. Maps over and onto finite dimensional vector spaces. Associated matrix. Computation of basis for the image and the kernel of a linear map.

Fundamental theorem.

Composition of linear maps. Ring of endomorphisms. Inverse of an isomorphism. Matrix associated with a composition.. Basis change in a vector space. Connection of base change with the matrix associated with a linear map.

The aim is to solve some problems on linear maps defined over infinite dimensional vector spaces, although focus shall be primarily on the finite dimensional case. Using the associated matrix and the theory of vector spaces and linear systems, basis for the kernel and image shall be obtained.

Problems on the composition of linear maps. Computation of the matrix associated with the inverse map of an isomorphism.

Specific objectives:

Introduce the fundamental concepts and relate them to the mathematical contents covered in other courses. Acquaint the student with vectors other than the familiar examples deriving from physical applications. Relate the injectivity and surjectivity of a linear map to its kernel and image.

Justify the correspondence between invertible and bijective linear maps. Relate the composition of linear maps to the product of their associated matrices. Computation of basis and analysis of their properties. Highlight the theory of base change, which shall be paramount in subsequent units: pay special attention to the relation between vector components in different basis and base vectors.

Relate new concepts to the student's general background: vector spaces, matrix properties, resolution of linear systems, rang of a vector system and implicit equations of a subspace.

Relate the contents of this topic to the properties they already know about matrices. . Insist on geometric applications.

Full-or-part-time: 16h 48m

Theory classes: 3h

Practical classes: 4h

Self study : 9h 48m

5. Euclidean space

Description:

Bilinear forms. Examples and basic properties. Matrix associated to a bilinear form. Changing the base. Symmetric bilinear form. Quadratic form. Definite forms. Canonical form and normal form of a real symmetric bilinear form.

Problems are solved by reduction of a symmetric bilinear form to its canonical form and normal. Change of basis.

Definition of inner product. Examples. Basic properties. Orthogonal subspace. Orthogonal and orthonormal basis. Orthogonal projection. Pythagoras theorem and law of the parallelepiped. Fourier coefficient. Schwarz inequality, Bessel and triangular.

Method of Gram-Schmidt. Geometric interpretations.

Properties of inner product. Orthogonal projection onto a subspace. Geometric interpretations.

Specific objectives:

To develop the properties of bilinear forms, especially the symmetrical, preparing its subsequent application in Calculus.

Working properties of symmetric bilinear forms using the method of elementary row operations they already know.

Present the definitions and general properties and continually interpret in real Euclidean space of three dimensions with which the student is familiar. It aims at an abstract knowledge of Euclidean space applications are working especially geometry.

Acquiring skill in use of abstract properties. General properties apply to real three-dimensional Euclidean space.

Full-or-part-time: 19h 12m

Theory classes: 4h

Practical classes: 4h

Self study : 11h 12m

6. Endomorphism and matrix reduction

Description:

Eigenvalues and eigenvectors. Characteristic polynomial. Diagonalization general theorem.

Diagonalization problems.

Elementary diagonalization theorem. Examples. Triagonalization basic theorem. Examples. Cayley-Hamilton. Examples and applications.

Triangulation problems.

Specific objectives:

Being able to diagonalize a matrix facilitates the obtaining of its basic properties and its manipulation. This will be crucial in applications.

Full-or-part-time: 19h 12m

Theory classes: 4h

Practical classes: 4h

Self study : 11h 12m

7. Operators and spectral theorems

Description:

Definitions and basic properties. Transposed. Associated matrices. Normal operators. Properties. Spectral theorem.

Problems of normal operators and properties of normal matrices.

Real normal operators. Symmetric and orthogonal operators. Spectral theorem for symmetric operators. Geometric interpretations.

Real normal operator problems.

Specific objectives:

Most matrices that appear in engineering are symmetric matrices. It should be clear that these are diagonally orthogonal.

Geometric applications of orthogonal matrices are also essential.

Full-or-part-time: 19h 12m

Theory classes: 4h

Practical classes: 4h

Self study : 11h 12m

GRADING SYSTEM

The final grade is obtained from partial qualifications as follows:

E0: Continuous assessment activities

E1: Test of the units developed on the first half of the academic term

E2: Test of the units developed on the second half of the academic term

E3: Global test of the course

The student has to choose whether to take the test E2 or E3

$$NF1 = 0.3E0 + 0.35E1 + 0.35E2$$

$$NF2 = 0.3E0 + 0.7E3$$

$$\text{Final Mark} = \max \{NF1, NF2\}$$

The exams consist of a part with questions on concepts associated with learning objectives in terms of subject knowledge or understanding, application and a set of exercises

Concerning to the grade E0, if there is the chance, some of the activities will be performed within the framework of the Engimath@UPC+ project.

Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation tests will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

These tests must be authorized by the corresponding Head of Studies, at the request of the professor responsible for the course, and will be carried out within the corresponding academic period.

EXAMINATION RULES.

Failure to perform a laboratory or continuous assessment activity in the scheduled period will result in a mark of zero in that activity.

BIBLIOGRAPHY

Basic:

- Rojo, J. Álgebra lineal. 2a ed. Madrid: McGrawHill, 2007. ISBN 978-84-481-5635-0.
- Rojo, J.; Martín, I. Ejercicios y problemas de álgebra lineal. 2a ed. Madrid: McGraw-Hill, 2005. ISBN 8448198581.
- Strang, G. Introduction to linear algebra. 6th ed. Wellesley: Cambridge Press, 2023. ISBN 9781733146678.

Complementary:

- Proskuriakov, I.V. 2000 problemas de álgebra lineal. Barcelona: Reverté, 1984. ISBN 8429151095.
- Hoffman, K.; Kunze, R. Linear algebra. 2nd ed. India: Pearson, 2015. ISBN 9789332550070.