# Course guide
# 2500012 - GECCVECTEQ - Vector Calculus and Differential Equation

**Last modified:** 01/10/2023

**Unit in charge:** Barcelona School of Civil Engineering
**Teaching unit:** 751 - DECA - Department of Civil and Environmental Engineering.

**Degree:** BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2020). (Compulsory subject).

**Academic year:** 2023 **ECTS Credits:** 6.0 **Languages:** English

## LECTURER

**Coordinating lecturer:** MATTEO GIACOMINI

**Others:** IRENE ARIAS VICENTE, MARINO ARROYO BALAGUER, DAVID CODONY GISBERT, PEDRO DIEZ MEJIA, SONIA FERNANDEZ MENDEZ, MATTEO GIACOMINI

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

**Specific:**
14392. Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge about: linear algebra; geometry; differential geometry; differential and integral calculation; differential equations and partial derivatives; numerical methods; numerical algorithmic; Statistics and optimization. (Basic training module)

## TEACHING METHODOLOGY

The course consists of 2 hours per week of classroom activity (large size group) and 2 hours weekly with half the students (medium size group).

The 2 hours in the large size groups are devoted to theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises.

The 2 hours in the medium size groups is devoted to solving practical problems with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENEA: content, program of learning and assessment activities conducted and literature.

Although most of the sessions will be given in the language indicated, sessions supported by other occasional guest experts may be held in other languages.

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## LEARNING OBJECTIVES OF THE SUBJECT

Integrals along curves. Integrals over surfaces. The integral theorems of vector analysis. Introduction and general features of PDEs. The Heat equation. Laplace's and Poisson's equations. The wave equation. Classification of second order equations. The weak form

1 Ability to relate differential equations in ordinary with engineering problems.
2 Ability to program simple solutions through basic software and to obtain numerical solutions.
3 Ability to develop solutions to these problems under simple conditions that allow an analysis of these solutions, including a parametric study.

Knowledge of ordinary differential equations. Basic knowledge of differential equations in partial derivatives, types, some analytical solutions in particular cases of special interest in engineering. Integrals along curves (Parametrized curves. The path integral, arc length. Change of parametrization for path integrals. Path integrals along paths defined on a parametrized surface. Line integrals. Physical interpretation: Work done by force fields, circulation. Change of parametrization for line integrals. Line integrals of gradient fields).
Integrals over surfaces (Parametrized surfaces. The integral of a scalar function over a parametrized surface. Change of parametrization. Surface integral of vector fields. Orientation. Change of parametrization. Physical interpretation: Flux.)
The integral theorems of vector analysis (Divergence and curl, and their physical interpretation and properties. Vector identities. Gradient, divergence and curl in cylindrical and spherical coordinates. Simple and elementary regions and their boundaries. Green's Theorem. Vector forms of Green's theorem and the divergence theorem in the plane. Stoke's theorem. The curl as circulation per unit area. Applications: Faraday's law, falling cats. Conservative fields. Gauss' divergence theorem. Divergence as flux per unit volume. Application: conservation of energy and the derivation of the heat equation. )
Introduction and general features of PDEs (Definition. Examples from mathematical physics. Linear PDE (superposition, homogeneous). Basic tools: divergence theorem).
The Heat equation (Derivation from heat flow balance. Heat conduction in 1D rod. Dimensionless scaling. Solution by separation of variables. Properties of solution. Thermal boundary conditions. The weak maximum principle. Non-homogeneous problems).
Laplace's and Poisson's equations (Equilibrium problems. Well-posedness. Dimensionless scaling. Solution by separation of variables. Laplace's equation inside a rectangle. Laplace equation inside a circular disk. Fluid flow outside a circular cylinder (lift). Qualitative properties of Laplace's equation. Mean value properties. Maximum/minimum principles. Uniqueness of solutions. Poisson's equation. Solution by separation of variables).
The wave equation (Derivation for a vertically vibrating string. Boundary conditions. Solution by separation of variables. Vibrating membrane. Some well-posed problems. Stability. First order wave equation. System of first order wave equations. Method of characteristic lines).

Classification of second order equations
Review of three important examples of PDEs. Motivating example. A general framework. Domain dependence and general features of PDEs
The weak form
Derivation of the weak form of Laplace's equation and the heat equation.

This module presents an introduction to vector calculus and integral calculus over curves and surfaces. The approach is multidisciplinary, with the aim of developing skills in the use of basic mathematical tools for the modeling and analysis of physical problems described by partial differential equations.

## STUDY LOAD

| Type | Hours | Percentage |
|---|---|---|
| Self study | 84,0 | 56.00 |
| Hours medium group | 30,0 | 20.00 |
| Guided activities | 6,0 | 4.00 |
| Hours large group | 30,0 | 20.00 |

**Total learning time:** 150 h

## CONTENTS

### Modelling with PDEs

**Description:**
1D heat equation
Heat equation in multiple dimensions
Derivation of the heat equation using integral theorems

**Full-or-part-time:** 14h 23m
Practical classes: 6h
Self study : 8h 23m

### Integral along lines

**Description:**
Parametrised curves
Arc length and reparametrisation
Path integrals and line integrals
Examples and exercises

**Full-or-part-time:** 19h 12m
Theory classes: 4h
Practical classes: 2h
Laboratory classes: 2h
Self study : 11h 12m

### Integral on surfaces

**Description:**
Parametrised surfaces
Surface integrals
Integrals of scalar and vectorfields
Examples and exercises

**Full-or-part-time:** 19h 12m
Theory classes: 4h
Practical classes: 2h
Laboratory classes: 2h
Self study : 11h 12m

### Vector calculus

**Description:**
Review of differential operators
Examples and exercises

**Full-or-part-time:** 9h 36m
Practical classes: 2h
Laboratory classes: 2h
Self study : 5h 36m

## Intergal theorems

**Description:**
Green's theorem
Stokes' theorem
Application: Faraday's law
Gauss' theorem
Application: mass conservation
Examples and exercises

**Full-or-part-time:** 28h 47m
Theory classes: 6h
Practical classes: 2h
Laboratory classes: 4h
Self study : 16h 47m

## Heat equation

**Description:**
Existence and uniqueness
Separation of variables
Non-homogeneous case
Examples and exercises

**Full-or-part-time:** 24h
Theory classes: 2h
Practical classes: 4h
Laboratory classes: 4h
Self study : 14h

## Laplace equation

**Description:**
Derivation of the Laplace equation
Separation of variables
Examples and exercises

**Full-or-part-time:** 14h 23m
Theory classes: 2h
Practical classes: 2h
Laboratory classes: 2h
Self study : 8h 23m

## Wave equation

**Description:**
Derivation of the wave equation
Method of characteristics
Examples and exercises

**Full-or-part-time:** 14h 23m
Theory classes: 2h
Practical classes: 2h
Laboratory classes: 2h
Self study : 8h 23m

## GRADING SYSTEM

The grade for the subject is obtained from the two partial exams and from a continuous assessment during the module.

The continuous evaluation consists of several activities, both individual and group, of incremental training, carried out during the module, both inside and outside the classroom.

The final grade will be computed as follows:
- 35% written exam on the first part of the module (partial exam 1);
- 35% written exam of the second part of the module (partial exam 2);
- 30% class work and homework to deliver.

The assessment tests consist of questions on concepts associated with learning objectives integrated into a set of application exercises.

## EXAMINATION RULES.

The assignments must be sent through ATENeA respecting the announced deadline. Late submissions or assignments submitted by other means will not be accepted and will be graded 0.

Assignments must be completed individually: students are encouraged to confront each other over issues, but submitted work must be the result of their own efforts of each students. Plagiarism in homework will be penalized with a 0 in the class work grade.

The written exams (test 1 and 2) must be taken individually and books or class notes will not be accepted. Plagiarism during exams will be penalized with a 0 in the final grade for the subject.

Students who fail the ordinary exams will have the option to take a reassessment test within the period set by the academic calendar. Students who have already passed the subject will not be able to take the reassessment test.

The maximum grade in the case of taking the reassessment exam will be five (5.0). The non-attendance of a student summoned to the reassessment test, held within the set period, may not lead to another test at a later date.

## BIBLIOGRAPHY

**Basic:**
- Marsden, J.E; Tromba, A.J. Vector Calculus. New York: WH Freeman, 2012. ISBN 9781429224048.
- Haberman, R. Applied partial differential equations with Fourier series and boundary value problems. Boston: Pearson, 2019. ISBN 9780134995434.