# Course guide
# 270202 - ALG - Algebra

**Last modified:** 19/07/2023

**Unit in charge:** Barcelona School of Informatics
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** BACHELOR'S DEGREE IN DATA SCIENCE AND ENGINEERING (Syllabus 2017). (Compulsory subject).

**Academic year:** 2023 **ECTS Credits:** 7.5 **Languages:** Catalan

## LECTURER

**Coordinating lecturer:** ANA RIO DOVAL

**Others:** Primer quadrimestre:
JOSEP ELGUETA MONTO - 11, 12
ANA RIO DOVAL - 11, 12

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

**Specific:**
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.

**Generical:**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.

**Transversal:**
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.

**Basic:**
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

## TEACHING METHODOLOGY

Different methodologies will be considered for lectures and exercises classes.
The lectures will consist mainly of master classes, based on presentations and explanations on the slate; the problem classes will be to solve exercises and practice concepts learned in the theory sessions.
Both of them may incorporate examples o short projects using python or similar software.

## LEARNING OBJECTIVES OF THE SUBJECT

1.Acquisition of the basic knowledge of linear algebra (vector spaces, matrices, linear systems)
2.Recognize concepts of linear algebra within interdisciplinary problems.
3.Learn how to use linear algebra in solving problems of data analysis and modeling.
4.Using linear algebra tools in mathematical problems
5.Using software to solve exercises related to linear algebra
6.Understanding of the notions of matrix decomposition, its geometric interpretation and its application in exercise solving

## STUDY LOAD

| Type | Hours | Percentage |
|---|---|---|
| Self study | 112,5 | 60.00 |
| Hours small group | 30,0 | 16.00 |
| Hours large group | 45,0 | 24.00 |

**Total learning time:** 187.5 h

## CONTENTS

### The real coordinate space

**Description:**
Vectors. Dot product (scalar product). Norm. Angle. Linear independence. Bases. Gram-Schmidt. Coordinate system. Points. Distance. Angle.

### Linear Maps

**Description:**
Linear maps. Matrices. Kernel and image. Systems of linear equations. Gaussian elimination. Subspaces. Invertible matrices. Change of basis. Endomorphisms and automorphisms

### Vector subpaces

**Description:**
Vector subspaces. Bases. Intersection and sum. Orthogonal complement. Orthogonal Projection.

### Diagonalization

**Description:**
Eigenvalues and eigenvectors; characteristic polynomial; algebraic and geometric multiplicity, diagonalization criteria, application to the computation of power of matrices and functions of matrices. Special case of Markov matrices and symmetric matrices. Spectral Theorem.

### Projections. Isometries.

**Description:**
Matriu d'una projecció. Classificació d'isometries en dimensions 2 i 3.

### Linear discrete dynamical systems

**Description:**
Madelling of problems via linear discrete dynamical systems, resolution and analysis of particular and generic solutions; long term behaviour of the solutions; numerical methods for the computation of eigenvalues and eigenvectors. Power iteration. Perron-Frobenius Theorem. Recurrencies.

### Applications

**Description:**
Singular value decomposition; matrix norms; application to rank approximation and dimensionality reduction in data and image analysis. Pseudoinvers and least squares. Errors.

## ACTIVITIES

### Development of topic 1

**Specific objectives:**
1, 2, 5

**Related competencies :**
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 17h
Theory classes: 4h
Practical classes: 3h
Self study: 10h

### Development of topic 2

**Specific objectives:**
1, 2, 3, 4, 5

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 17h
Theory classes: 4h
Practical classes: 3h
Self study: 10h

**Development of topic 3**

**Specific objectives:**
1, 2, 3, 4, 5

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 33h
Theory classes: 9h
Practical classes: 6h
Self study: 18h

**Development of topic 4**

**Specific objectives:**
1, 2, 3, 4, 5

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 17h
Theory classes: 4h
Practical classes: 3h
Self study: 10h

**Development of topic 5**

**Specific objectives:**
1, 2, 3, 4, 5, 6

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 32h
Theory classes: 9h
Practical classes: 6h
Self study: 17h

**Development of topic 6**

**Specific objectives:**
1, 2, 3, 4, 5, 6

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 20h
Theory classes: 5h
Practical classes: 3h
Guided activities: 2h
Self study: 10h

**Development of topic 7**

**Specific objectives:**
1, 2, 3, 4, 5, 6

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 33h 30m
Theory classes: 10h
Practical classes: 6h
Self study: 17h 30m

**Final exam**

**Description:**
Final exam

**Specific objectives:**
1, 2, 3, 4, 6

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 8h
Guided activities: 3h
Self study: 5h

**Partial exam**

**Description:**
Partial exam

**Specific objectives:**
1, 2, 3, 4, 6

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.
CB1. That students have demonstrated to possess and understand knowledge in an area of ??study that starts from the base of general secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply Knowledge from the vanguard of their field of study.

**Full-or-part-time:** 6h 30m
Guided activities: 1h 30m
Self study: 5h

**Avaluation of problem resolution using Python or another software**

**Specific objectives:**
3, 4, 5

**Related competencies :**
CG2. Choose and apply the most appropriate methods and techniques to a problem defined by data that represents a challenge for its volume, speed, variety or heterogeneity, including computer, mathematical, statistical and signal processing methods.
CE1. Skillfully use mathematical concepts and methods that underlie the problems of science and data engineering.
CT6. Autonomous Learning. Detect deficiencies in one's own knowledge and overcome them through critical reflection and the choice of the best action to extend this knowledge.
CT5. Solvent use of information resources. Manage the acquisition, structuring, analysis and visualization of data and information in the field of specialty and critically evaluate the results of such management.

**Full-or-part-time:** 3h 30m
Guided activities: 1h
Self study: 2h 30m

## GRADING SYSTEM

The assessment of the subject will consist of the marks: P, F, L
The mark P will be obtained from the partial exam.
The mark F will be obtained from the final exam.
The mark L will be obtained by evaluation of problem resolution using python or another software.
The final mark will be computed as follows:

Note = maximum (60% F + 30% P + 10% L, F)

The re-evaluation grade will be the mark of the reavaluation exam.

## BIBLIOGRAPHY

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- Poole, D. Linear algebra: a modern introduction. 4th. Stamford: Cengage Learning, 2015. ISBN 9781285463247.
- Meyer, C.D. Matrix analysis and applied linear algebra. Philadelphia: SIAM, Society for Industrial and Applied Mathematics, 2000. ISBN 9780898714548.
- Aubanell, A.; Benseny, A.; Delshams, A. Eines bàsiques de càlcul numèric: amb 87 problemes resolts. Bellaterra: Universitat Autònoma de Barcelona, 1991. ISBN 8479292318.

## RESOURCES

**Hyperlink:**
- https://docs.sympy.org/latest/tutorial/matrices.html- https://numpy.org/doc/stable/reference/