UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Course guide
300018 - ALA - Linear Algebra and Applications

Last modified: 01/06/2023

Unit in charge: Castelldefels School of Telecommunications and Aerospace Engineering
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN NETWORK ENGINEERING (Syllabus 2009). (Compulsory subject).
BACHELOR'S DEGREE IN TELECOMMUNICATIONS SYSTEMS ENGINEERING (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: Definit a la infoweb de l'assignatura.

Others: Definit a la infoweb de l'assignatura.

PRIOR SKILLS

Calculus and Mathematics for Telecommunications
The ability to carry out arithmetic calculations and simplify algebraic expressions.
Familiarity with the concept of a function and the graphic representation of a function.
The ability to apply differential and integral calculus of one or more variables.
Familiarity with the Laplace transform.
Capacity for abstract thinking.

REQUIREMENTS

Upper secondary school mathematics.
Students must have taken or be taking Calculus and Mathematics for Telecommunications.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE 1 TELECOM. Students will acquire the ability to solve mathematical problems for engineering. An aptitude for applying knowledge of linear algebra, geometry, differential geometry, differential and integral calculus, differential equations and partial differential equations, numerical methods, numerical algorithms, statistics and optimisation. (CIN/352/2009, BOE 20.2.2009)

Transversal:
2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.
3. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
4. TEAMWORK - Level 1. Working in a team and making positive contributions once the aims and group and individual responsibilities have been defined. Reaching joint decisions on the strategy to be followed.

TEACHING METHODOLOGY

Two sessions lasting 90 minutes each per week in which theoretical concepts are dealed and example problems are solved. These sessions combine expository and participatory (cooperative learning) models.

One session lasting an hour per week is devoted to solving problems using a software tool such as Wiris or Maple. It also introduces the basic concepts that students will work on in the directed activities.

One hour per week of directed activities in which students present, analyse and solve practical cases analytically and numerically (using the software mentioned above). Problem-/project-based learning methods are used to carry out the directed activities. There are two types of directed activity sessions. In the first type, students will work individually on a guide downloaded from Atenea that contains a background study. Students will also hand in a questionnaire before carrying out the laboratory assignment. The second type of directed activity, a continuation of the first, will be carried out in the laboratory. Students will be expected to use mathematical software to solve a linear algebra problem following written guidelines. Directed activities will be conducted under the guidance of the lecturer. The reduced size of the groups allows the shortcomings of the previous activity to be detected and enhances students' understanding of course topics. The lecturer can also provide individual guidance. This is also beneficial for the more able students. The lecturer's proximity ensures continuous assessment.

Self-directed learning hours must be devoted to studying the course content and doing the exercises set by the lecturer.

LEARNING OBJECTIVES OF THE SUBJECT

On completion of Linear Algebra and Applications, students will:

- Understand basic algebraic structures, Boolean algebra and modular arithmetic.
Be able to solve linear systems of equations.
- Be able to carry out operations with matrices.
- Be familiar with the properties of vector spaces.
- Understand linear applications, changes of basis and matrix diagonalisation.
- Be able to carry out operations with scalar products and manipulate bases. Be familiar with orthonormalisation.
- Be able to geometrically interpret and solve the most common first-order differential equations, linear differential equations of order n and systems of first-order linear differential equations with constant coefficients. Be able to find particular solutions.
- Be able to use a computer tool (Wiris, Matlab or equivalent) to solve linear algebra problems.

STUDY LOAD

Type	Hours	Percentage
Hours small group	13,0	8.67
Hours large group	39,0	26.00
Guided activities	14,0	9.33
Self study	84,0	56.00

Total learning time: 150 h

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CONTENTS

Algebraic structures

Description:
1.1 Binary operations and properties. Algebraic structures (semigroups, groups, rings, fields).
1.2 Whole numbers: order of operation. Divisibility: the fundamental theory of arithmetic. Congruence: divisibility tests.
1.3 Reticles and Boolean algebra. Logical circuits.
1.4 Polynomials.

Related activities:
Test 1, level test 1, directed activities 1, 2, 3 and 4

Full-or-part-time: 36h
Theory classes: 9h
Laboratory classes: 3h
Guided activities: 4h
Self study : 20h

Linear systems of equations, matrices and determinants

Description:
2.1 Matrices. Operations with matrices. Inverse matrices. Rank. Gauss method.
2.2 Determinants.
2.3 Linear equation systems. Discussion and solution of systems. Cramer's rule. The superposition principle.

Related activities:
Level test 1; Directed activities 5 and 6

Full-or-part-time: 23h
Theory classes: 6h
Laboratory classes: 2h
Guided activities: 2h
Self study : 13h

Vector spaces

Description:
3.1 Vector spaces and subspaces. Subspace generated by a set: linear combinations. Linear dependence and independence. Generator systems.
3.2 Bases. Dimension. Coordinates of a vector basis. Change of basis.
3.3 Operations with subspaces: direct sum.

Related activities:
Test 2; Level test 2; Directed activities 7 and 8

Full-or-part-time: 23h
Theory classes: 6h
Laboratory classes: 2h
Guided activities: 2h
Self study : 13h

Linear operators and diagonalisation

Description:
4.1 Definitions and properties. Kernel and image. Matrix of a linear application. Change of basis in linear applications.
4.2 Diagonalisable endomorphisms and matrices. Eigenvectors and eigenvalues. Characteristic polynomial.
4.3 Diagonalisation. Cayley-Hamilton theorem. First decomposition theorem.
4.4 Scalar product. Orthogonal and orthonormal bases. Orthogonal projection.

Related activities:
Control 2; Level test 2; Directed activities 9, 10, 11 and 12

Full-or-part-time: 34h
Theory classes: 9h
Laboratory classes: 3h
Guided activities: 3h
Self study : 19h

Differential equations

Description:
5.1 First-order differential equations Definition. Separable, linear and homogeneous equations. Exact differential equations.
5.2 Higher-order linear differential equations with constant coefficients. Test method for obtaining a particular solution in the inhomogeneous case.
5.3 Linear differential systems of equations with constant coefficients. Substitution method. Homogenous and inhomogeneous systems. Application of the Laplace transform.

Related activities:
Test 2; Level test 3; Directed activities 13 and 14

Full-or-part-time: 34h
Theory classes: 9h
Laboratory classes: 3h
Guided activities: 3h
Self study : 19h

ACTIVITIES

TEST 1

Description:
Individual test. Two or three exercises similar to the ones in the list of problems worked on in class.

Specific objectives:
For students to work with linear operators and diagonalisation and calculate eigenvalues and eigenvectors.

Material:
Lecture notes and list of the problems available on the digital campus

Delivery:
Test
Assessment: See section on tests.

Full-or-part-time: 10h 45m
Theory classes: 0h 45m
Self study: 10h

TEST 2

Description:
Individual test. Two or three exercises similar to the ones in the list of problems worked on in class.

Specific objectives:
For students to work with linear operators and diagonalisation and calculate eigenvalues and eigenvectors.

Material:
Lecture notes and list of the problems available on the digital campus

Delivery:
Test
Assessment: See section on tests.

Full-or-part-time: 10h 45m
Theory classes: 0h 45m
Self study: 10h

DIRECTED ACTIVITIES 1 AND 2 (INTRODUCTION TO MATHEMATICAL SOFTWARE)

Description:
Introduction to free and University-licensed software and how it might be used in the course. The software should allow students to operate with matrices and solve linear systems of equations. (e.g.

Specific objectives:
To assess students' familiarity with basic sets of instructions in linear algebra software.

Material:
Materials AD2 and AD3 (available on the digital campus)

Delivery:
Assignment 1: Completed questionnaire.
Assignment 2: Methodological design and completed assignment.
Assessment: See the section on group assignments.

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

DIRECTED ACTIVITIES 3 AND 4 (MODULAR ARITHMETIC AND RSA ENCRYPTION)

Description:
The background study describes the mathematics of RSA encryption. Students will complete and hand in a questionnaire at the beginning of the second directed activity, which is carried out in the laboratory under the guidance of the lecturer. The directed activity consists in designing the methodology and solving a practical RSA public key encryption exercise using the concepts and methods contained in the background study.

Specific objectives:
For students to apply modular arithmetic to RSA encryption

Material:
Materials AD3 and AD4 (available on Atenea)

Delivery:
Assignment 3: Completed questionnaire.
Assignment 4: Methodological design and completed assignment.
Assessment: See section on group assignments

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

DIRECTED ACTIVITIES 13 AND 14 (DIFFERENTIAL EQUATIONS)

Description:
The background study involves revising systems of equations and their applications to predator-prey models. Students hand in a questionnaire at the beginning of the second directed activity, which is carried out in the laboratory under the guidance of the lecturer and involves designing a methodology and solving problems such as an adapted predator-prey race model. (sections 11.20 and 11.21 of Google's 2005 annual report)

Specific objectives:
Applying numerical methods for solving differential equations to solve prey-predator models (for example propagation of agents in networks, etc.)

Material:
Materials AD13 and AD14 (available on Atenea)

Delivery:
Assignment 13: Completed questionnaire.
Assignment 14: Methodological design and completed assignment.
Assessment: See section on group assignments

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

DIRECTED ACTIVITIES 5 AND 6 (MATRICES AND NETWORKS)

Description:
The background study describes the relationship between matrices and networks (adjacency matrix). Students hand in a questionnaire at the beginning of the second directed activity, which is carried out in the laboratory under the guidance of the lecturer. Students use real and simulated network data to carry out practical network property calculations for the matrix involving hubs and distances and operations. They also consider matrix products and column sums.

Specific objectives:
For students to apply matrix operations to the study of networks.

Material:
Materials AD5 and AD6 (available on Atenea)

Delivery:
Assignment 5: Completed questionnaire.
Assignment 6: Methodological design and completed assignment.
Assessment: See section on group assignments

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

DIRECTED ACTIVITIES 7 AND 8 (VECTOR SPACES AND ERROR-CORRECTING CODES)

Description:
The background study describes the relationship between vector spaces and matrices and linear error-correcting codes. Students hand in a questionnaire at the beginning of the second activity, which is carried out in the laboratory under the guidance of the lecturer and involves designing the methodology and solving practical cases of coding-decoding.

Specific objectives:
For students to apply vector spaces and matrix operations to the study of linear error-correcting codes.

Material:
Materials AD7 and AD8 (available on Atenea)

Delivery:
Assignment 7: Completed questionnaire.
Assignment 8: Methodological design and completed assignment.
Assessment: See section on group assignments

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

DIRECTED ACTIVITIES 9 AND 10 (DIAGONALISATION)

Description:
The background study describes methods for finding eigenvalues and eigenvectors. Students hand in a questionnaire at the beginning of the second activity, which is carried out in the laboratory under the guidance of the lecturer. The activity involves designing a methodology and carrying out eigenvalue and eigenvector calculations (e.g. Google's PageRank algorithm and sections 11.15 and 11.17 of its 2005 annual report).

Specific objectives:
Diagonalisation. Power method. Relation to matrix power. Applications (e.g. PageRank and agent status).

Material:
Materials AD9 and AD10 (available on Atenea)

Delivery:
Assignment 9: Completed questionnaire.
Assignment 10: Methodological design and completed assignment.
Assessment: See section on group assignments

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

DIRECTED ACTIVITIES 11 AND 12 (LINEAR OPERATORS)

Description:
The background study shows the uses and scope of affine transformations (plane linear transformations) in fractal image compression. Students hand in a questionnaire at the beginning of the second activity, which is carried out in the laboratory under the guidance of the lecturer and involves solving specific cases of fractal compression (section 11.14 of Google's 2005 annual report).

Specific objectives:
Apply affine transformations for fractal image compression.

Material:
Materials AD11 and AD12 (available on Atenea)

Delivery:
Assignment 11: Completed questionnaire.
Assignment 12: Methodological design and completed assignment.
Assessment: See section on group assignments

Full-or-part-time: 4h
Guided activities: 2h
Self study: 2h

GRADING SYSTEM

The assessment criteria defined in the course infoweb will be applied.

EXAMINATION RULES.

Tests are held during theory-based lectures and last approximately 20 minutes.
The first examination is sat halfway through the semester (the specific week is set by the School).
The second examination is sat a week after classes have ended.
The examinations last 90 minutes. The same examination in set for all the groups.
Two individual level tests are taken on Atenea.
Four of the directed activity assignments will be assessed.

BIBLIOGRAPHY

Basic:

- Lay, David C.; Murrieta Murrieta, Jesús Elmer; Alfaro Pastor, Javier. Álgebra lineal y sus aplicaciones [on line]. 3a. México [etc.]: Pearson Educación, 2007 [Consultation: 04/10/2023]. Available on: http://www.ingebook.com/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=6765. ISBN 9789702609063.
- Braun, Martin. Ecuaciones diferenciales y sus aplicaciones. México, D.F.: Grupo Editorial Iberoamérica, 1990. ISBN 9687270586.

Complementary:

- Williams, Gareth; Hano Roa, Ma. del Carmen. Álgebra lineal con aplicaciones. 4a. México [etc.]: McGraw-Hill, 2002. ISBN 970103838X.
- Marcellán, Francisco; Casasus, Luis; Zarzo, Alejandro. Ecuaciones diferenciales : problemas lineales y aplicaciones. Madrid, [etc.]: McGraw-Hill, 1990. ISBN 8476155115.
- Anton, Howard; Rorres, Chris. Elementary linear algebra: applications version. New York: John Wiley & Sons, 2005. ISBN 9780471669593.

RESOURCES

Other resources:

Material available on the digital campus (Atenea):

1) Specific materials for the cooperative learning (Jigsaw) sessions in three sections (roles):
2) Lecture notes.
3) Sets of problems.
4) Background study and questionnaire for the first group of directed activities and script and questionnaire for the second group.