



## Course guide

# 300203 - CAL-A - Calculus (Aeronautical Engineering)

Last modified: 16/11/2023

**Unit in charge:** Castelldefels School of Telecommunications and Aerospace Engineering  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** BACHELOR'S DEGREE IN AEROSPACE SYSTEMS ENGINEERING (Syllabus 2015). (Compulsory subject).

**Academic year:** 2023    **ECTS Credits:** 6.0    **Languages:** Catalan, Spanish

### LECTURER

**Coordinating lecturer:** Definit a la infoweb de l'assignatura.

**Others:** Definit a la infoweb de l'assignatura.

### PRIOR SKILLS

Upper secondary school mathematics.

The ability to work with abstract concepts.

Familiarity with the concept of a function and the graphic representation of a function.

The ability to perform mathematical calculations, simplifications of algebraic expressions and calculus of elementary functions of one variable.

Basic understanding of geometry.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

#### Specific:

CE1. CE 1 AERO. Capacidad para la resolución de los problemas matemáticos que puedan plantearse en la ingeniería. Aptitud para aplicar los conocimientos sobre: álgebra lineal; geometría; geometría diferencial; cálculo diferencial e integral; ecuaciones diferenciales y en derivadas parciales; métodos numéricos; algorítmica numérica; estadística y optimización. (CIN/308/2009, BOE 18.2.2009)

#### General:

CG1. (ENG) CG1 - Capacidad para el diseño, desarrollo y gestión en el ámbito de la ingeniería aeronáutica que tengan por objeto, de acuerdo con los conocimientos adquiridos, los vehículos aeroespaciales, los sistemas de propulsión aeroespacial, los materiales aeroespaciales, las infraestructuras aeroportuarias, las infraestructuras de aeronavegación y cualquier sistema de gestión del espacio, del tráfico y del transporte aéreo.

CG2. (ENG) CG2 - Planificación, redacción, dirección y gestión de proyectos, cálculo y fabricación en el ámbito de la ingeniería aeronáutica que tengan por objeto, de acuerdo con los conocimientos adquiridos, los vehículos aeroespaciales, los sistemas de propulsión aeroespacial, los materiales aeroespaciales, las infraestructuras aeroportuarias, las infraestructuras de aeronavegación y cualquier sistema de gestión del espacio, del tráfico y del transporte aéreo.

#### Transversal:

CT6. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

CT4. TEAMWORK - Level 1. Working in a team and making positive contributions once the aims and group and individual responsibilities have been defined. Reaching joint decisions on the strategy to be followed.



**Basic:**

CB1. (ENG) CB1 - Que los estudiantes hayan demostrado poseer y comprender conocimientos en un área de estudio que parte de la base de la

educación secundaria general, y se suele encontrar a un nivel que, si bien se apoya en libros de texto avanzados, incluye también algunos aspectos que implican conocimientos procedentes de la vanguardia de su campo de estudio

CB2. (ENG) CB2 - Que los estudiantes sepan aplicar sus conocimientos a su trabajo o vocación de una forma profesional y posean las competencias que suelen demostrarse por medio de la elaboración y defensa de argumentos y la resolución de problemas dentro de su área de estudio

CB3. (ENG) CB3 - Que los estudiantes tengan la capacidad de reunir e interpretar datos relevantes (normalmente dentro de su área de estudio)

para emitir juicios que incluyan una reflexión sobre temas relevantes de índole social, científica o ética

CB4. (ENG) CB4 - Que los estudiantes puedan transmitir información, ideas, problemas y soluciones a un público tanto especializado como no especializado

CB5. (ENG) CB5 - Que los estudiantes hayan desarrollado aquellas habilidades de aprendizaje necesarias para emprender estudios posteriores con un alto grado de autonomía

## TEACHING METHODOLOGY

In the theory lectures the fundamental concepts of the subject will be introduced, and basic exercises and problem solving techniques will be presented.

In the problems sessions, exercises and problems proposed a priori by the lecturer and autonomously prepared by the students will be discussed and solved.

## LEARNING OBJECTIVES OF THE SUBJECT

At the end of the Calculus course, students has to be able to:

- Understand the concept of limit of a function at a point and know some calculation techniques.
- Solve problems involving derivatives of functions of one variable.
- Know Taylor's formula and its application to the local study and the approximate evaluation of functions.
- Understand the concept of integral, and calculate areas of regions in the plane, and volumes of some solids in space.
- Know some primitive calculation techniques.
- Identify conics and quadrics from their equations.
- Acquire skills in calculations involving curves and surfaces.
- Understand and know how to geometrically interpret the concepts of directional derivative, differential and gradient.
- Calculate local and absolute extrema of functions of one variables, and conditioned extrema of functions of several variables.

## STUDY LOAD

Type	Hours	Percentage
Hours medium group	13,0	8.67
Hours large group	39,0	26.00
Guided activities	14,0	9.33
Self study	84,0	56.00

**Total learning time:** 150 h



## CONTENTS

### Content 1: Introduction. Functions

**Description:**

- 1.1 Functions of one variable: definition, domain, elementary functions.
- 1.2 Limits, continuity, asymptotes.
- 1.3 Conics: circle, parabola, ellipse, hyperbola.

**Related activities:**

Activities 1, 3, and 4.

**Full-or-part-time:** 23h

Theory classes: 4h

Practical classes: 4h

Guided activities: 1h

Self study : 14h

### Content 2: Derivation of functions of one variable

**Description:**

- 2.1 Concept of derivative: definition, geometric interpretation, tangent and normal straight lines..
- 2.2 Calculation of derivatives: the derivative function, the chain rule, logarithmic derivation, implicit derivation.
- 2.3 Limits: the criterion of L'Hôpital, the compression criterion.
- 2.4 Approximation of functions: Taylor's polynomial, Lagrange residue.
- 2.5 Extrema of a function: relative extrema, absolute extrema, optimization problems.

**Related activities:**

Activities 1, 3, and 4.

**Full-or-part-time:** 37h 30m

Theory classes: 7h

Practical classes: 6h

Guided activities: 1h 30m

Self study : 23h

### Content 3: Integration of functions of one variable

**Description:**

- 3.1 Indefinite integral: primitive of a function, concept of indefinite integral, geometric interpretation.
- 3.2 Calculation of primitives: immediate integrals, linearity of the indefinite integral, almost immediate integrals, integration by change of variable, integration by parts, integration of rational functions, integration of trigonometric and hyperbolic functions, integration of some irrational functions.
- 3.3 Definite integral: definition, properties of the definite integral, the Fundamental Theorem of Calculus, Barrow's rule, change of variable in the definite integral.
- 3.4 Applications of the definite integral: calculation of areas of planar figures, calculation of volumes of solids of revolution.
- 3.5 Improper integrals.

**Related activities:**

Activities 2, 3, and 4.

**Full-or-part-time:** 31h 30m

Theory classes: 6h

Practical classes: 5h

Guided activities: 1h 30m

Self study : 19h



#### Content 4: Functions of several variables

**Description:**

- 4.1 Scalar functions.
- 4.2 Functions of  $R^2$  on  $R^2$ : domain, range, contours and sections, surfaces in implicit form, quadrics.
- 4.3 Functions of  $R^3$  on  $R$ . Level surfaces.
- 4.4 Functions of  $R^n$  on  $R$ . Level sets.
- 4.5 Vector functions.
- 4.6 Parameterized curves.
- 4.7 Tangent vector to a parameterized curve.

**Related activities:**

Activities 2 and 4.

**Full-or-part-time:** 26h

Theory classes: 5h

Practical classes: 4h

Guided activities: 1h

Self study : 16h

#### Content 5: Differential calculus on $R^n$

**Description:**

- 5.1 Introduction.
- 5.2 Directional derivatives, partial derivatives, gradient.
- 5.3 Tangent plane.
- 5.4 Differentiability of scalar functions. The chain rule.
- 5.5 Properties of the gradient. Applications
- 5.6 Differentiability of vector functions. Jacobian matrix
- 5.7 Second partial derivatives. Schwarz's theorem.
- 5.8 Conditional extrema of scalar functions: boundary, bounded sets, compacts, Weierstrass' theorem, algorithm for calculating conditional extrema, the parameterization method, Lagrange multipliers.

**Related activities:**

Activity 4.

**Full-or-part-time:** 32h

Theory classes: 6h

Practical classes: 5h

Guided activities: 0h 30m

Self study : 20h 30m

## ACTIVITIES

#### Activity 1: Test C1

**Description:**

Test of Content 1 and first part of Content 2.

**Full-or-part-time:** 1h

Guided activities: 1h



### Activity 2: test C2

**Description:**

Test of the second part of Content 3 and Content 4.

**Full-or-part-time:** 1h

Guided activities: 1h

### Activity 3: Mid term exam

**Description:**

Exam of Contents 1, 2, and first part of 3.

**Full-or-part-time:** 1h 30m

Guided activities: 1h 30m

### Activity 5: Final term exam

**Description:**

Exam of Contents 1, 2, 3, 4 and 5.

**Full-or-part-time:** 2h

Guided activities: 2h

## GRADING SYSTEM

Defined at the subject's infoweb.

## EXAMINATION RULES.

Tests are made during lecture sessions and are previously announced in ATENEA. The mid and final semester exams are conducted according to the EETAC's schedule.

Tests and exams are made individually. The use of books, notes, calculator, software and mobile phone is not allowed.

## BIBLIOGRAPHY

**Basic:**

- "Diferenciación". Marsden, Jerrold E.; Tromba, Anthony. Cálculo vectorial [on line]. Madrid [etc.]: Addison Wesley, 2004. P. 75-188 [Consultation: 26/07/2022]. Available on: [https://www-ingobook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=7634](https://www-ingobook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7634). -
- "Funciones con valores vectoriales". Marsden, Jerrold E.; Tromba, Anthony. Cálculo vectorial [on line]. Madrid [etc.]: Addison Wesley, 2004. P. 189-240 [Consultation: 26/07/2022]. Available on: [https://www-ingobook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=7634](https://www-ingobook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7634). - Larson, Ron; Edwards, Bruce H.; Ibarra Escutia, Joel. Cálculo. Vol. 2, Cálculo 2, de varias variables. 9a. México [etc.]: McGraw-Hill, 2010. ISBN 9789701071342.
- Larson, Ron; Edwards, Bruce H.; Ibarra Escutia, Joel. Cálculo. Vol. 1, Cálculo 1, de una variable [on line]. 9a. México [etc.]: McGraw-Hill, 2010 [Consultation: 26/07/2022]. Available on: [https://www-ingobook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=5686](https://www-ingobook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=5686). ISBN 9786071502735.

**Complementary:**



- Salas, Saturnino L.; Hille, Einar; Etgen, Garret J. Calculus : una y varias variables [on line]. 4<sup>a</sup>. Barcelona [etc.]: Reverté, 2002  
[Consultation: 26/07/2022]. Available on:  
<https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/reader.action?docID=5635414>. ISBN 9788429151565.
- Barrière, Lali. Fonaments matemàtics per a l'enginyeria de telecomunicació. Barcelona: Edicions UPC, 2007. ISBN 9788483019078.

## RESOURCES

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### Other resources:

- Course schedule with syllabus.
- Initial knowledge material.
- Slides of the contents of the course.
- List of exercises (and solutions) of the course.
- Sample models of exams and tests of previous courses.

All of them are available in ATENEA.