



## Course guide

# 300242 - AM2 - Further Mathematics 2

Last modified: 01/06/2023

**Unit in charge:** Castelldefels School of Telecommunications and Aerospace Engineering  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** BACHELOR'S DEGREE IN AEROSPACE SYSTEMS ENGINEERING (Syllabus 2015). (Compulsory subject).

**Academic year:** 2023    **ECTS Credits:** 7.5    **Languages:** Catalan, Spanish

### LECTURER

**Coordinating lecturer:** Definit a la infoweb de l'assignatura.

**Others:** Definit a la infoweb de l'assignatura.

### PRIOR SKILLS

Basic Probability and combinatorics. Derivation and integration of functions in one and more variables, Resolution of first order ordinary differential equations. Matrices, Numerical series. Eigenvectors and eigenvalues determination.

### REQUIREMENTS

Have passed or be enrolled in 1A: Algebra and Geometry and Calculus and 1B Further Mathematics

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

#### Specific:

CE1. CE 1 AERO. Capacidad para la resolución de los problemas matemáticos que puedan plantearse en la ingeniería. Aptitud para aplicar los conocimientos sobre: álgebra lineal; geometría; geometría diferencial; cálculo diferencial e integral; ecuaciones diferenciales y en derivadas parciales; métodos numéricos; algorítmica numérica; estadística y optimización. (CIN/308/2009, BOE 18.2.2009)

#### General:

CG1. (ENG) CG1 - Capacidad para el diseño, desarrollo y gestión en el ámbito de la ingeniería aeronáutica que tengan por objeto, de acuerdo con los conocimientos adquiridos, los vehículos aeroespaciales, los sistemas de propulsión aeroespacial, los materiales aeroespaciales, las infraestructuras aeroportuarias, las infraestructuras de aeronavegación y cualquier sistema de gestión del espacio, del tráfico y del transporte aéreo.

#### Transversal:

CT6. SELF-DIRECTED LEARNING - Level 2: Completing set tasks based on the guidelines set by lecturers. Devoting the time needed to complete each task, including personal contributions and expanding on the recommended information sources.

CT3. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 2. Using strategies for preparing and giving oral presentations. Writing texts and documents whose content is coherent, well structured and free of spelling and grammatical errors.



**Basic:**

CB1. (ENG) CB1 - Que los estudiantes hayan demostrado poseer y comprender conocimientos en un área de estudio que parte de la base de la

educación secundaria general, y se suele encontrar a un nivel que, si bien se apoya en libros de texto avanzados, incluye también algunos aspectos que implican conocimientos procedentes de la vanguardia de su campo de estudio

CB2. (ENG) CB2 - Que los estudiantes sepan aplicar sus conocimientos a su trabajo o vocación de una forma profesional y posean las competencias que suelen demostrarse por medio de la elaboración y defensa de argumentos y la resolución de problemas dentro de su área de estudio

CB3. (ENG) CB3 - Que los estudiantes tengan la capacidad de reunir e interpretar datos relevantes (normalmente dentro de su área de estudio)

para emitir juicios que incluyan una reflexión sobre temas relevantes de índole social, científica o ética

CB5. (ENG) CB5 - Que los estudiantes hayan desarrollado aquellas habilidades de aprendizaje necesarias para emprender estudios posteriores con un alto grado de autonomía

## TEACHING METHODOLOGY

There are four 1-hour theory sessions per week. In theory sessions, theoretical concepts are worked out and illustrative exercises are solved. These sessions combine the expository model with the participatory one.

In problem sessions, exercises from the list of problems are solved encouraging problem solving by the students through a more personalized attention.

In the lab sessions, specific software is used to solve problems that require a computer. Students work in a small group on some problems (AD or supervised activities) that may require the use of a computer and which will be delivered at the end.

Frequent and personalized feedback is given to each student, through marking and comments on laboratory work, supervised activities, controls and exams and the publication of grades on the Digital Campus.

## LEARNING OBJECTIVES OF THE SUBJECT

The student must be able to:

Distinguish between random and deterministic experiences.

Apply counting techniques.

Distinguish between continuous random variables and discrete random variables. Distinguish the different random variables introduced in the syllabus. Calculate probabilities for each of the random variables.

Determine confidence intervals for the mean and standard deviation of a population. Perform linear regressions from data.

Introduce the theory of graphs and networks and study basic algorithms.

Basic study of stochastic processes and queueing theory.

Manipulate a statistical data package to simulate simple random experiences.

Determine the error made in approximating a function by the Taylor polynomial.

Calculate and manipulate series of powers of elementary functions.

Derive numerically by the proposed methods. Solve ordinary differential equations and partial derivatives using the proposed numerical methods.

Calculate basic properties of graphs. Know and apply some basic algorithms on graphs and networks (PageRank, Kruskal, Prim, Dijkstra, Christofides)

## STUDY LOAD

Type	Hours	Percentage
Guided activities	30,5	16.27
Self study	105,0	56.00
Hours large group	52,0	27.73

**Total learning time:** 187.5 h



## CONTENTS

X

### Description:

Ordered, unordered samples, with and without replacement. Combinatorial numbers and properties.

### Specific objectives:

Mathematical formulation and ability to calculate how many different combinations or options are obtained with different methods of selecting elements from a given set.

### Related activities:

Supervised activity 1. Control 1

### Full-or-part-time: 20h

Theory classes: 6h

Laboratory classes: 2h

Guided activities: 2h

Self study : 10h

## Probability

### Description:

Probabilities in a finite sample space. Conditional probability. Independent events. Bayes theorem.

### Specific objectives:

Mathematical meaning of probability and ability to determine the probability that a given event occurs, when random experiments are performed.

### Related activities:

Supervised activity 1. Control 1

### Full-or-part-time: 20h

Theory classes: 6h

Laboratory classes: 2h

Guided activities: 2h

Self study : 10h

## Probability distributions

### Description:

Random variables. Expected value and variance.

Distributions for discrete random variables: Binomial, Geometric and Poisson. Distributions for continuous random variables: Uniform, Exponential and Normal.

### Specific objectives:

Knowledge of the most relevant probability distributions. Application to specific problems.

### Related activities:

Supervised activity 2. Minitab lab session. Control 1

### Full-or-part-time: 20h

Theory classes: 6h

Laboratory classes: 2h

Guided activities: 2h

Self study : 10h



X

**Description:**

Random samples. Estimators.  
Confidence intervals for the mean and standard deviation of a population.

**Specific objectives:**

Knowledge and application of sampling techniques and estimators. Determination of confidence intervals.

**Related activities:**

Supervised activity 2. Minitab lab session. Control 1

**Full-or-part-time:** 10h

Theory classes: 3h  
Laboratory classes: 1h  
Guided activities: 1h  
Self study : 5h

**Linear regression.**

**Description:**

Simple linear model. Estimation by least squares. Correlation

**Specific objectives:**

Determine correlations in linear models.

**Related activities:**

Supervised activity 2. Midterm exam.

**Full-or-part-time:** 9h 30m

Theory classes: 2h 30m  
Laboratory classes: 1h  
Guided activities: 1h  
Self study : 5h

**Queueing theory**

**Description:**

Introduction to Poisson stochastic processes. Exponential queues M/M/ with finite and infinite population. Mean time in the system, in the queue and service time. Mean number of users in the system, in the queue and being serviced. Equations of the queue in stationary regime. Probabilities of the states in stationary regime.

**Specific objectives:**

Knowledge and applications of stochastic processes and queuing theory.

**Related activities:**

Supervised activity 2. Mid term exam.

**Full-or-part-time:** 16h

Theory classes: 4h  
Laboratory classes: 1h 30m  
Guided activities: 1h 30m  
Self study : 9h



### (ENG) Taylor formula. Power series.

**Description:**

Approximation of a function by using Taylor polynomials and study of the errors.  
Lagrange remainder and orders of magnitude. Power series. Radius of convergence.  
Hadamard's formula.

**Specific objectives:**

Study the convergence of a power series and its sum. Control the error when truncating a power series.

**Related activities:**

Supervised activity 3. Control 2

**Full-or-part-time:** 16h

Theory classes: 4h  
Laboratory classes: 1h  
Guided activities: 1h  
Self study : 10h

### (ENG) Numerical methods for differential equations.

**Description:**

(ENG) Numerical differentiation. Initial-value problems for ordinary differential equations. Runge-Kutta methods.  
Numerical solutions for partial differential equations (PDEs). Numerical methods for elliptic, parabolic, and hyperbolic second-order linear PDEs.

**Specific objectives:**

Numerical methods to solve ordinary differential equations and partial differential equations.

**Related activities:**

Supervised activity 3. Control 2.

**Full-or-part-time:** 54h 30m

Theory classes: 14h  
Laboratory classes: 3h  
Guided activities: 3h 30m  
Self study : 34h

### Graph theory and algorithms

**Description:**

Graphs. Basic definitions: vertices, edges, arcs, degrees of the vertices, digraphs, multigraphs, paths, cycles, circuits, trees, weighted graphs and digraphs.  
Algorithmic complexity. Minimum spanning tree of a weighted graph (Prim's, Prim-Jarnik and Kruskal Algorithms). Shortest paths (Dijkstra and Floyd's algorithm). Shortest eulerian path (Euler's Theorem). Shortest hamiltonian paths (Christofides' Approximative Algorithm and algorithm of exponential complexity using dynamic programming. Ford-Fulkerson's Algorithm.

**Specific objectives:**

Basic knowledge on graph theory, networks and algorithms.

**Related activities:**

Supervised activity 4. Final exam.

**Full-or-part-time:** 21h 30m

Theory classes: 6h 30m  
Laboratory classes: 1h 30m  
Guided activities: 1h 30m  
Self study : 12h



## ACTIVITIES

### Control 1

**Description:**

Control on counting techniques and probability. Solving exercises similar to those in the lists worked in class.

**Specific objectives:**

Verify learning process

**Delivery:**

15% in final mark

**Full-or-part-time:** 1h

Theory classes: 1h

### Control 2

**Description:**

Control on numerical methods. Resolution of exercises similar to those worked in class.

**Specific objectives:**

Verify

**Delivery:**

15% of final mark.

**Full-or-part-time:** 1h

Theory classes: 1h

### Minitab lab session.

**Description:**

Minitab lab session. Simulations of random experiences. Most usual probability distributions. Simulations of random experiments: Samples and Estimation.

**Specific objectives:**

Learn Minitab instructions needed to work out the results seen in problem classes. Simulation of random experiments with software.

**Full-or-part-time:** 3h

Laboratory classes: 1h 30m

Self study: 1h 30m



### Supervised activites 1,2,3,4.

**Description:**

Solve in a small group one or more problems previously worked individually and following the script of the corresponding AD.

**Specific objectives:**

Solve in a small group one or more problems previously worked individually and following the script of the corresponding AD.

**Material:**

List of solved exercices, Guide for the AD.

**Delivery:**

A report on each directed activity. Total weight in the final grade 10%

**Full-or-part-time:** 10h

Laboratory classes: 5h

Self study: 5h

### Midterm exam

**Description:**

Assessment of the first part of the course. (25% of the overall grade)

**Specific objectives:**

Verify the level of achievement of the subjects studied.

**Delivery:**

Written exam.

**Full-or-part-time:** 1h 30m

Theory classes: 1h 30m

### Final exam

**Description:**

Assessment of the first part of the course. (25% of the overall grade)

**Specific objectives:**

Verify the level of achievement of the subjects studied.

**Delivery:**

Written exam.

**Full-or-part-time:** 1h 30m

Theory classes: 1h 30m

## GRADING SYSTEM

Evaluation criteria is defined elsewhere.

## EXAMINATION RULES.

All activities are mandatory. Exams and controls will be carried out individually.



## BIBLIOGRAPHY

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### Basic:

- Burillo, Josep; Miralles, Alícia; Serra, Oriol. Probabilitat i estadística [on line]. Barcelona: Edicions UPC, 2003 [Consultation: 15/04/2020]. Available on: <http://hdl.handle.net/2099.3/36808>. ISBN 8483016869.
- Burden, Richard L.; Faires, J. Douglas. Numerical analysis. 9th. [Pacific Grove (California), etc.]: Brooks/Cole Cengage Learning, 2011. ISBN 9780538735643.
- Gimbert i Quintilla, Joan. Apropament a la teoria de grafs i als seus algorismes [on line]. [Lleida] : [Zaragoza]: Edicions de la Universitat de Lleida ; F.V. Libros, 1998 [Consultation: 27/10/2022]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6776292>. ISBN 8489727651.

### Complementary:

- Lipschutz, Seymour; Schiller, John J.; Cortiñas Vázquez, Pedro; Santos Peña, Julián; Muñoz Alamillos, Ángel; Guzmán Justicia, Luis. Introducción a la probabilidad y estadística. Madrid [etc.]: McGraw-Hill/Interamericana de España, 2001. ISBN 8448125045.
- Estrada, Ernesto; Knight, Philip. A first course in network theory. 1a ed.. New York: Oxford University Press, 2015. ISBN 9780198726463.