

# Course guide 320049 - RM - Strength of Materials

**Last modified:** 19/04/2023

Unit in charge: Terrassa School of Industrial, Aerospace and Audiovisual Engineering

**Teaching unit:** 712 - EM - Department of Mechanical Engineering.

Degree: BACHELOR'S DEGREE IN MECHANICAL ENGINEERING (Syllabus 2009). (Compulsory subject).

Academic year: 2023 ECTS Credits: 6.0 Languages: Catalan, Spanish

#### **LECTURER**

Coordinating lecturer: Javier Alvarez del Castillo

Others: Albert Catalan

# **DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES**

#### Specific:

1. MEC: Students will acquire the skills and knowledge necessary to apply the fundamentals of the elasticity and strength of materials to the behaviour of real solids.

#### Transversal:

2. TEAMWORK - Level 3. Managing and making work groups effective. Resolving possible conflicts, valuing working with others, assessing the effectiveness of a team and presenting the final results.

# **TEACHING METHODOLOGY**

- · Face-to-face lecture sessions.
- · Face-to-face practical sessions.
- · Independent study and exercises.
- · Cooperative learning.

In the lecture sessions, the lecturer introduces each topic's theoretical foundations, concepts, methods and results, providing illustrative examples to facilitate comprehension.

There are four types of face-to-face practical sessions:

- a) Sessions in which the lecturer guides students in the analysis of data and the application of problem-solving techniques, concepts and theoretical results.
- b) Sessions in which group work is presented.
- c) Cooperative learning sessions.
- d) Evaluation sessions.

Students are expected to study independently in order to grasp the concepts and solve the assigned exercises, either alone or with the help of a computer.

# **LEARNING OBJECTIVES OF THE SUBJECT**

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# **STUDY LOAD**

Туре	Hours	Percentage
Hours medium group	30,0	20.83
Hours large group	30,0	20.83
Self study	84,0	58.33

Total learning time: 144 h

# **CONTENTS**

#### **TOPIC 1: PRISMATIC MEMBERS**

#### **Description:**

- 1.1 Purpose and utility of strength of materials.
- 1.2 Prismatic members: concept and types.
- 1.3 External forces.
- 1.3.1 Directly applied forces.
- 1.3.2 Interfacial adhesion strengths.
- 1.4 Statically determinate and statically indeterminate systems.
- 1.5 Assumptions made.
- 1.6 Limitations of beam theory.
- 1.7 Calculation process.
- 1.8 Application exercises.

# Specific objectives:

This topic serves as an introduction to strength of materials, and is also the first of several topics covering the basics of prismatic members. We explore various concepts related to prismatic members, including their various types, interfacial adhesion, and the concept of static determinacy and indeterminacy. We emphasise the importance of the starting hypothesis in relation to prismatic members and explore the limitations of prismatic members. Finally, we study the calculation process followed when one works with prismatic members.

Although the exercises included in this topic could be considered a review of the statics topics covered in the subject Mechanics and Theory of Mechanisms I, they place special emphasis on the calculation of reactions with various types of supports under different cases of loads and geometries, as well as on the calculation of the degree of static indeterminacy.

Full-or-part-time: 7h Theory classes: 4h Self study: 3h

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# **TOPIC 2: STRESS ON CROSS-SECTIONS**

# **Description:**

- 2.1 Stress on a cross-section of a prismatic member.
- 2.2 State of stress in a cross-section. Equivalence equations.
- 2.3 Laws of stress and corresponding diagrams.
- 2.4 Special case a prismatic member subjected to a load on its middle plane.
- 2.5 Equilibrium of cross-sections.
- 2.6 Application exercises.

# Specific objectives:

In this topic, we explore the various kinds of stress that can act on a cross-section of a prismatic member¿ axial force, shear force, bending moment and torsional moment¿and learn how to determine each of them when all external forces are known, establishing the relevant laws and diagrams for statically determinate systems. We will also study the equilibrium of cross-sections.

As part of this topic, students will complete exercises involving the calculation and representation of free-body diagrams (axial force, bending moment, torsional moment, shear force) for prismatic members with different geometries and different types of loads (forces and moments with different orientations, distributed and uniform loads) and different types of support. Special attention will be given to methodology, sign convention and the study of stresses on cross-sections in equilibrium. Through exercises involving free-body diagrams, we will also explore the relationship between shear force and the slope of the bending moment diagram.

**Full-or-part-time:** 8h Theory classes: 2h Practical classes: 2h Self study: 4h



#### **TOPIC 3: Normal state in cross-sections**

#### **Description:**

- 3.1 Normal stresses caused by normal force and bending moment.
- 3.2 Neutral axis.
- 3.3 Strain energy on an elementary cross-section.
- 3.4 Relative motion of the faces of a cross-section.
- 3.5 Application exercises.

#### Specific objectives:

In this topic, we study the normal stress state in cross-sections of prismatic members. We analyse the normal stresses produced by axial force and bending moment. We introduce the concept of the neutral axis, as well as the expression of the relative motion of the faces of an elementary cross-section, starting with the assumption of conservation of plane sections or the Navier¿Bernoulli hypothesis and applying Hooke's law. We study the statement of the generalised Navier¿Bernoulli principle (for cases in which bending occurs along with shear force or torsional moment). Students begin to familiarise themselves with the general case, combined bending, symmetrical bending and the special cases of each. Discussion of this subject will continue in Topics 5 and 6. The exercises for this topic will enable students to:

- · Understand the assumptions made in the calculation of normal stresses in cross-sections of prismatic members and, therefore, the inherent limitations of the mathematical model.
- · Calculate normal stresses in cross-sections of prismatic members subjected to bending moment and normal force, and understand the reasons behind the mathematical calculation.
- $\cdot$  Comprehend the principle of superposition as applied to stresses (bending moment and normal force, in this case) as well as its impact on the stress distribution along a cross-section.
- · Understand stress distribution on the basis of the effect that stress has on a cross-section and identify the fibres experiencing zero stress (neutral axis), the relative maxima, the area under tension and the area under compression.
- · Reflect on and calculate the relative motion of the faces of a differential cross-section, fibre elongation and unit strain in particular directions, and strain energy in a cross-section subjected to stress, as a means of laying the groundwork for the calculation of elastic strain and deformation, as well as the practical use of energy methods.
- $\cdot$  Understand the relationships between external stress, unit strain and elongation, as well as the expressions for calculating them.
- · Understand the superposition principle as applied to stresses and their effects, and reflect on and calculate cases of combined stress, such as general combined bending, symmetrical bending, and special cases of each.

**Full-or-part-time:** 8h Theory classes: 2h Practical classes: 2h Self study: 4h



# **TOPIC 4: Stress and strain caused by normal force**

#### **Description:**

Stress and strain caused by normal force.

- 4.1 Stress and strain in a cross-section subjected to a normal force. Strain energy.
- 4.2 Application to the calculation of straight members.
- 4.3 Stress and strain in a member under its own weight.
- 4.4 Bodies of equivalent tensile strength.
- 4.5 Statically indeterminate systems consisting of elements performing simple compression or tension.
- 4.6 Erection stress and thermal stress.
- 4.7 Application exercises.

# Specific objectives:

In this topic, we explore the effects of the axial force and its application to the calculation of straight members. We analyse the stresses and strains in a bar under its own weight and define bodies of equal strength. We discuss statically indeterminate systems consisting of elements under forces of this type, as well as the erection and thermal stresses that can occur in systems of this sort.

The exercises for this topic will enable students to:

- · Understand the effects brought about by a normal force on a cross-section of a prismatic member by applying the theoretical knowledge acquired in the last two topics to models consisting of real members and real loads, such as the straight members subjected to normal force, bars under their own weight, members with a variable cross-section under normal force and constant tensile strength, and members subjected to erection and thermal stresses.
- · Become familiar with techniques for calculating simple statically indeterminate systems.

**Full-or-part-time:** 8h Theory classes: 2h Practical classes: 2h Self study: 4h

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# **TOPIC 5: Stress and strain caused by bending force**

# **Description:**

- 5.1 Symmetric pure bending.
- 5.1.1 Navier's formula.
- 5.1.2 Neutral axis.
- 5.1.3 Section design and modulus.
- 5.1.4 Deformation of cross-sections.
- 5.1.5 Geometric efficiency of a section.
- 5.2 Biaxial bending.
- 5.2.1 Stress.
- 5.2.2 Neutral axis.
- 5.2.3 Section design.
- 5.2.4 Deformation of cross-sections.
- 5.3 Application exercises.

#### Specific objectives:

In this topic, we explore the effects of bending moment (introduced in Topic 3 with the assumption of conservation of plane sections, in the case of pure bending, and the generalised Navier¿Bernoulli principle) for the cases of symmetric pure bending and biaxial bending.

The exercises for this topic will enable students to:

- · Understand the effects of bending moment on a cross-section of a prismatic member by applying knowledge acquired in Topic 3.
- $\cdot$  Understand and calculate stress distribution on the basis of the effect that bending moment (both pure and biaxial) has on a cross-section and identify the fibres experiencing zero stress (neutral axis), the relative maxima, the area under tension and the area under compression.
- · Identify, in the calculation of the state of stress in a cross-section, the role of moment of inertia.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Self study: 6h



# **TOPIC 6: Combined bending**

# **Description:**

- 6.1 Combined bending. General case.
- 6.2 Relationship between the centre of pressure and the neutral axis.
- 6.3 Central core of a section. Determination of the central core.
- 6.4 Symmetrical combined bending.
- 6.5 Combined bending in materials without tensile strength.
- 6.6 Application exercises.

# Specific objectives:

In this topic, we cover in greater detail the combination of various types of stresses (axial force and bending moment) that go into combined bending. We study the various positions of the neutral axis according to the location of the centre of pressure and introduce the concept of the central core of a section. We also analyse the special cases of symmetric combined bending and combined bending in materials without tensile strength.

The exercises for this topic will enable students to:

- $\cdot$  Gain an understanding of the effects of axial force and bending moment in two of the principal axes of inertia in a cross-section by analysing the state of stress generated in combined bending, the concept of centre of pressure and central core, and their effect on materials without tensile strength.
- · Calculate and represent states of stress in a cross-section due to combined bending.
- $\cdot$  Calculate and represent the centre of pressure, the neutral axis and the central core in a cross-section subjected to combined bending in simple geometries. Reflect on the progression of cracks in cross-sections of structures consisting of materials without tensile strength.

Full-or-part-time: 10h Theory classes: 2h Practical classes: 2h Self study: 6h



#### TOPIC 7: Simple bending: stress and strain caused by shear force

#### **Description:**

- 7.1 Stress caused by shear force in simple bending.
- 7.2 Basic formula for calculating shear stress.
- 7.3 Elementary theory. Application to different types of cross-section.
- 7.4 Element strain caused by shear force. Strain energy.
- 7.5 Principal stresses in simple bending.
- 7.6 Influence of shear force on the design of a structure subjected to simple bending.
- 7.7 Shear stresses in thin sections.
- 7.8 Sections in which the main axis is not the axis of symmetry. Shear centre.
- 7.9 Cases of pure shear calculation:
- 7.9.1 Bolts and screws.
- 7.9.2 Bolted or riveted joints.
- 7.9.3 Welded joints.
- 7.10 Calculation of stresses at joints in compound beams.
- 7.11 Application exercises.

#### Specific objectives:

In this topic, we explore the stress and strain caused by shear force in simple bending. We also cover the combined action of shear force and bending moment, and analyse the influence of shear force on the design of a structure subjected to simple bending. We study shear stresses in thin sections and sections in which the main axis is not the axis of symmetry, introducing, in the latter case, the concept of shear centre. We also explore the special case of pure shear. Although pure shear occurs only rarely, there are many cases that can be solved using this concept (bolts, screws, rivets, pins, etc.). As for welded joints, we consider only the case in which the weld faces the same direction as the force; more complex cases are covered in the technology subject. Finally, we explore an interesting application involving joints in compound beams.

The exercises for this topic will enable students to:

- $\cdot$  Study the combined effect of shear force and bending moment on a cross-section of a prismatic member.
- $\cdot$  Understand the limitations of the calculation model. Understand the conditions under which the effect of shear force is relevant in the design of structures subjected to simple bending.
- · Understand in detail the effect of simple bending on thin sections.
- · Understand the effect of simple bending by applying equations for calculating the design of bolts, screws, bolted joints, riveted joints and welded joints, as well as in the calculation of composite beams.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Self study: 6h



#### **TOPIC 8: TORSION**

#### **Description:**

- 8.1 Torsion. General information.
- 8.2 Elementary theory of torsion in beams with circular cross-sections.
- 8.3 Strain energy.
- 8.3.1 Element strain in cross-sections.
- 8.4 Torsion in beams with non-circular cross-sections.
- 8.5 Beams with rectangular cross-sections.
- 8.6 Torsion in thin-walled open tubes.
- 8.7 Torsion in thin-walled closed tubes.
- 8.8 Application exercises.

### Specific objectives:

In this topic, we study the effect of torsion on cross-sections. We begin by defining the elementary theory of beams with circular cross-sections before moving on to beams with other kinds of cross-sections, applying the theory of elasticity.

The exercises for this topic will enable students to:

- · Calculate the effect on the torsional stress state in beams with circular and annular cross-sections.
- · Gain an understanding of the limits of the elementary theory of torsion.
- · Calculate the effect of torsion on the stress state in beams with rectangular cross-sections.
- · Calculate the effect of torsion on the stress state in thin-walled closed tubes.
- · Become familiar with the calculation of shafts by studying beams with circular cross-sections subjected to torsion. As part of this topic, students will complete exercises that combine the use of failure criteria, the factor of safety, allowable stress, geometry and material.

### **TOPIC 9: COMPOUND STRESS WITH TORSION**

# Description:

- 9.1 Compound stress in general.
- $9.2\ \mbox{Calculation}$  of circular shafts subjected to bending and torsion.
- 9.3 Torsion combined with bending, normal force and shear force.
- 9.4 Coil springs.
- 9.5 Application exercises.

# Specific objectives:

In this topic, we explore different types of compound stress involving torsion. First, we cover bending combined with torsion and its application to the calculation of circular shafts, before moving on to the more general case of possible combinations; we then cover torsion combined with bending, with shear force and with axial force, one application of which is the case of coil springs. This topic is clearly of interest due to the summary provided of the various cases.

The exercises for this topic will enable students to:

- · Understand and calculate, as a means of summarising the previous topics, the state of stress in a cross-section of a prismatic member subjected to bending moment, torsional moment, axial moment and shear force; identify the fibre under the greatest or most unfavourable state of stress; and, using the most appropriate failure criterion, calculate the equivalent stress for comparison with the allowable stress in each circumstance.
- $\cdot$  Calculate combined stress involving torsional moment. Calculate shafts and coil springs, leaving aside, for the moment, issues of material fatigue.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Self study: 6h



# TOPIC 10: STUDY OF STRAIN IN PRISMATIC MEMBERS (I): NAVIER¿BRESSE EQUATIONS

# **Description:**

- 10.1 General Navier¿Bresse equations.
- 10.1.1 General case of warped prismatic members.
- 10.1.2 Case of a prismatic member subjected to a load on its middle plane.
- 10.2 Application exercises.

# Specific objectives:

Starting from element strain in cross-sections, we first obtain the general Navier¿Bresse equations and later derive specific equations for simpler cases.

The exercises for this topic will enable students to:

- · Learn to calculate rotations and displacements in a warped prismatic member and in a prismatic member subjected to a load on its middle plane, taking into account axial force, shear force, bending moment and torsional moment using the Navier¿Bresse equations.
- $\cdot$  Reflect on how displacement and deformed geometries are generated in prismatic members loaded with different kinds of forces, as well as the contribution of each type of force on the various sections of a prismatic member and an overview of these contributions.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Self study: 6h

# **TOPIC 11: STRAIN IN PRISMATIC MEMBERS**

# **Description:**

- 11.1 Strain in straight beams.
- 11.2 Elastic differential equation. Applications.
- 11.3 Calculation of rotations in supports. General case of a beam with moments at each end and the relative displacement of its supports.
- 11.4 Mohr's theorems. Applications.
- 11.5 Application exercises.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Self study: 6h



# TOPIC 12: STUDY OF STRAIN IN PRISMATIC MEMBERS (II): ENERGY METHODS

# Description:

- 12.1 Energy methods for calculating strain.
- 12.2 Expression of strain energy.
- 12.3 Application of Castigliano's theorem.
- 12.4 Application of the virtual work theorem.
- 12.5 Application exercises.

#### Specific objectives:

This is the last in the series of topics on strain. In this topic, we use the various energy theorems introduced in Topic 6 to calculate strain.

The exercises for this topic will enable students to:

- $\cdot$  Apply the energy theorems introduced in Topic 6 (Castigliano's theorem and the virtual work theorem) to the calculation of displacements and rotations.
- $\cdot$  Understand how the energy theorems simplify the calculation of displacements and rotations in cross-sections of prismatic members.
- · Calculate displacements in statically determinate frames and trusses.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Self study: 6h

# **TOPIC 13: STATICALLY INDETERMINATE SYSTEMS**

#### **Description:**

- 13.1 Types of statically indeterminate systems.
- 13.2 General approach to solving statically indeterminate systems.
- 13.2.1 Canonical equations.
- 13.2.2 Determination of influence lines.
- 13.3 Internally statically indeterminate systems.
- 13.3.1 Application of the Navier  $\grave{\epsilon} Bresse$  equations.
- 13.3.2 Application of the energy theorems.
- 13.4 Mohr's integral method.
- 13.5 Application exercises.

# Specific objectives:

In this first topic on static indeterminacy, we discuss statically indeterminate systems; define externally and internally statically indeterminate systems and explain how to determine their degree of static indeterminacy; study the force method of calculating statically indeterminate unknowns; analyse the various methods for solving statically indeterminate structures; and finally, discuss the use of symmetry properties in the calculation of statically indeterminate systems.

The exercises for this topic will enable students to:

- $\cdot \ \text{Practise using the methodology for solving internally and externally statically indeterminate systems.}$
- $\cdot$  Use the canonical equation method, the virtual work method and Castigliano's method to calculate statically indeterminate unknowns and displacement unknowns.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Laboratory classes: 6h



#### **TOPIC 14: SINGLE-SPAN STATICALLY INDETERMINATE BEAMS**

# **Description:**

- 14.1 Single-span statically indeterminate beams. Double fixed-end beam. Beam fixed at one end, supported at the other end.
- 14.2 Different calculation methods.
- 14.3 Strain in statically indeterminate beams.
- 14.4 Application exercises.

# Specific objectives:

As part of our analysis of continuous beams, we study the theorem of three moments. We examine the case of a beam with one fixed end and analyse the effect of relative settlement among supports, distinguishing between statically determinate and statically indeterminate elastic supports.

The following exercises will be carried out as part of this topic:

- · Application exercises involving the calculation of statically indeterminate reactions and the construction of free-body diagrams for single-span beams.
- $\cdot$  Application exercises involving the theorem of three moments for the calculation of statically indeterminate reactions and the construction of free-body diagrams for continuous beams.

**Full-or-part-time:** 10h Theory classes: 2h Practical classes: 2h Laboratory classes: 6h

# **GRADING SYSTEM**

First examination: 40% Second examination: 40%

Assessment of in-class work/theory: 20%

All those students who suspend want to improve note or can not attend the partial exam, will have the opportunity to examine the same day of the final exam. If the circumstances do not make it feasible that it is the same day as the final exam, the teacher responsible for the subject will propose, via the Atenea platform, that the aforementioned recovery test will be carried out another day, during class hours.

The new note of the recovery exam will replace the old one only in case it is higher.

For those students who meet the requirements and submit to the reevaluation examination, the grade of the reevaluation exam will replace the grades of all the on-site written evaluation acts (tests, midterm and final exams) and the grades obtained during the course for lab practices, works, projects and presentations will be kept.

If the final grade after reevaluation is lower than 5.0, it will replace the initial one only if it is higher. If the final grade after reevaluation is greater or equal to 5.0, the final grade of the subject will be pass 5.0.

# **EXAMINATION RULES.**

In-class examination.