UNIVERSITAT POLITĖCNICA
DE CATALUNYA
BARCELONATECH

## Course guide

## 340021 - FOMA-N1043 - Fundamentals of Mathematics

| Unit in charge: <br> Teaching unit: | Vilanova i la Geltrú School of Engineering <br> 749-MAT - Department of Mathematics. |
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| Degree: | BACHELOR'S DEGREE IN ELECTRICAL ENGINEERING (Syllabus 2009). (Compulsory subject). <br>  <br> BACHELOR'S DEGREE IN INDUSTRIAL DESIGN AND PRODUCT DEVELOPMENT ENGINEERING (Syllabus |
| 2009). (Compulsory subject). <br> BACHELOR'S DEGREE IN INDUSTRIAL ELECTRONICS AND AUTOMATIC CONTROL ENGINEERING (Syllabus <br> 2009). (Compulsory subject). <br> BACHELOR'S DEGREE IN MECHANICAL ENGINEERING (Syllabus 2009). (Compulsory subject). |  |
| Academic year: 2023 | ECTS Credits: $6.0 \quad$ Languages: Catalan |

## LECTURER

| Coordinating lecturer: | Prat Farran, Joana D'Arc |
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| Others: | Ponz Iranzo, Daniel |
|  | Prat Farran, Joana D'Arc |

## PRIOR SKILLS

To use and simplfy algebraic expressions.
To calculate and simplify function derivatives, using the basic rules of product, quocient and chaine rule.
To solve linear systems with Gauss and Cramer rule.
To operate with matrix (sum, product, invers and determinant).

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

## Specific:

1. CE1. Ability to solve arithmetic problems related to engineering. Aptitude to apply knowledge concerning: linear algebra, geometry, differential geometry, differential and integral calculus, differential and partial equations, numerical methods, numerical algorithms, statistics and optimization.

## Transversal:

2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

## TEACHING METHODOLOGY

The theory classes consist of theoretical explanations, description of examples and solution of selected problems, using various traditional and digital media.
In several parts of the subject, open source software or with UPC license will be used to solve applied problems.
The student will have non-attendance activities within the continuous evaluation.

## LEARNING OBJECTIVES OF THE SUBJECT

Use the fundamental tools of the differential calculus to study functions of one variable, and obtain approximations of functions through the Taylor polynomial

Understand the fundamental theorem of integration and its use in the solution of various problems.

Understand and know how to apply the basic numerical methods to calculate zeros and area calculus numerically. Understand machine precision and relative and absolute errors
and error propagation in basic operations.

Learn about the fundamental concepts and some examples of the use of vector spaces and linear applications.

Recognize a rotation matrix in space, calculate its axis of rotation and the angle.

Make use of specific software (matlab/Geogebra) to solve some problems.

STUDY LOAD

| Type | Hours | Percentage |
| :--- | :--- | :--- |
| Hours large group | 60,0 | 40.00 |
| Self study | 90,0 | 60.00 |

Total learning time: 150 h

## CONTENTS

## 1. Differential and integral calculus

## Description:

Real functions of a real variable: Study of continuity, study of derivability and calculation of the tangent line, calculation of relative extremes.
1 Review of elementary functions and functions defined in pieces.
2 Continuity. Bolzano theorem.
3 derivation. geometric interpretation of the derivative. Hôpital Rule.
4 Application to the local study of functions: Growth and decrease. Extremes
Real functions of a real variable: Calculation of integrals defined by change of variables and by parts. Barrow Rule
5 Fundamental theorem of algebra. Integral defined as area. Barrow's rule.
6 Integration defined by parts, change of variables.

## Specific objectives:

The student will be able to:
Study (and build on demand) the continuity and derivability of functions defined in pieces based on transferred elementary functions, etc.
Use theorems about continuous functions.
Calculate limits using the L'Hôpital rule in simple cases.
Explain the geometric meaning of the derivative at a point, and some examples of its use.
Study the local behavior of a function: growth and relative extremes (by hand and from the graph of the derived function). Know how to integrate by parts and by changes of variables.
State the fundamental concepts and results on integrals and interpret them in terms of areas delimited by Function graphs and the $x$ axis.
State the fundamental theorem of integral calculus and explain its importance in calculating definite integrals.
Know the integration by parts and the changes of variable.
Know how to calculate areas limited by two functions.

## Related activities:

Tasks 1, 2, 3
Full-or-part-time: 38 h
Theory classes: 18h
Self study : 20h

## 2. Numerical calculus

## Description:

Numerical calculus: errors and approximation, Taylor polynomial calculation and truncation error, numerical methods for calculating zeros and for calculating definite integrals. Use of Octave / Matlab.
1 Errors and approximation.
2 Taylor's polynomial. Residue.
3 Numerical methods for calculating zeros of functions: bisection method, of the falsi regula, of Newton-Raphson. Examples in Octave / Matlab environment.
4 Numerical methods for calculating definite integrals: trapezoid method, Simpson method. Examples in Octave / Matlab environment.

## Specific objectives:

Know the concept of absolute and relative error and propagation of errors.
Know the floating point representation
Know how to calculate how errors spread in basic operations
Calculate Taylor's polynomial of any order of a function at a point, and the shape of the residue.
Knowing how to use the bisection method, Newton-Raphson method (Octave functions) to numerically calculate zeros in an Octave / Matlab environment.
Know how to use the trapeze method and the Simpson method (Octave functions) to calculate numerically integral defined in an Octave / Matlab environment.
Recognize the exact decimals of the results provided by the numerical methods studied.
Be able to calculate zeros and areas with a fixed number of exact decimals.

## Related activities:

Tasks 1, 2, 3
Full-or-part-time: 36h
Theory classes: 18h
Self study : 18h

## 3. Linear algebra

## Description:

1. Vector spaces: To find out linear dependency/independency of vectors and calculate dimensions and basis of subspaces.
2. Linear maps: how to calculate the kernel and image and dimensions, its interpretation to solve linear systems. Rotations.
3. Use of linear algebra to geometric and model problems.

## Specific objectives:

The student will be able to:
Determine the dependence / linear independence of vectors.
Determine if a certain set of vectors is a vector subspace.
Calculate the dimension and a base of a vector subspace.
Determine whether an application between vector spaces is a linear application.
Calculate the matrix associated with a linear application in the canonical base.
Calculate the dimension and a base of the image and the kern of a linear application.
Use the range theorem to calculate image or kernel dimensions.
Calculate the properties of the linear system solutions in terms of linear application dimensions associated
Calculate the anti-image of a vector for a given linear application.
Use linear algebra to model and solve complex problems.

## Related activities:

Tasks 1, 2, 3
Full-or-part-time: 28h
Theory classes: 14h
Self study : 14h

## ACTIVITIES

## TASK 1

## Description:

Avaluation that can consist of practices with software (Matlab/Octave, Geogebra, ...), jobs or/and tests via platform ATENEA.
Full-or-part-time: 20h
Guided activities: 20h

## TASK 2

## Description:

Individual writed exam, the week of firsts exams, of contens given until then.
Full-or-part-time: $2 h$
Theory classes: 2 h

## TASK 3

## Description:

Individual writed exam of all Contents.
Full-or-part-time: 3h
Theory classes: 3h

## GRADING SYSTEM

A1 = mark Task 1, continuous evaluation (all content)
A2 = mark Task 2, attended exam (all content done until the middle period of exams)
A3 = mark Task 3, attended exam (all content)

The final mark is:

FINAL MARK $=0.3 * A 1+\max (0.2 * \mathrm{~A} 2+0.5 * \mathrm{~A} 3,0.7 * \mathrm{~A} 3)$
where all qualifications are calculated over 10.

Task A3 will be reevaluated.

## EXAMINATION RULES.

Under no circumstances may it be copied or allowed to be copied; failure to comply with this rule in any of the activities will result in a 0 in the grade for the corresponding activity or the final grade.
The particular regulations of the activities will be announced in each case with sufficient time.
All activities are individual.
The tasks A2 and A3 are on-site and individual and will be carried out in the week reserved for the first test period of the course and in the final evaluation period that appears in the academic Calendar respectively.
Activity A 1 is mix non-attendance and attendance.
The specific rules for each activity will be indicated in sufficient time.

## BIBLIOGRAPHY

## Basic:

- Amer Ramon, Rafael. Curs d'àlgebra lineal. 2a ed. Terrassa: Universitat Politècnica de Catalunya, 1998. ISBN 8484987841.
- Kreyszig, Erwin. Matemáticas avanzadas para ingeniería. 3a ed. México, D.F. [etc.]: Limusa, 2006. ISBN 9789681853113.
- Estela Carbonell, M. Rosa. Fonaments de càlcul. 2a ed. Barcelona: Edicions UPC, 2005. ISBN 8483018357.
- Anton, Howard. Introducción al álgebra lineal : con aplicaciones en negocios, economía, ingeniería, física, ciencias de la computación, teoría de la aproximación, ecología, sociología, demografía y genética. 5a ed. México, DF: Limusa, 2011. ISBN 9786070502903.


## Complementary:

- Stewart, James. Cálculo de una variable : trascendentes tempranas. 6a ed. México: International Thomson, 2008. ISBN 9789706866530.


## RESOURCES

## Other resources:

- Matlab online: https://es.mathworks.com/products/matlab-online.html />- OCtave: https://www.gnu.org/software/octave/ />Geogebra: https://www.geogebra.org/ />- Wolfram: http://mathworld.wolfram.com

