

Course guide

340025 - EQDI-F2O43 - Differential Equations

Last modified: 27/06/2023

Unit in charge: Vilanova i la Geltrú School of Engineering
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN ELECTRICAL ENGINEERING (Syllabus 2009). (Compulsory subject).
BACHELOR'S DEGREE IN INDUSTRIAL DESIGN AND PRODUCT DEVELOPMENT ENGINEERING (Syllabus 2009). (Optional subject).
BACHELOR'S DEGREE IN INDUSTRIAL ELECTRONICS AND AUTOMATIC CONTROL ENGINEERING (Syllabus 2009). (Compulsory subject).
BACHELOR'S DEGREE IN MECHANICAL ENGINEERING (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: Immaculada Massana Hugas

Others: Francesc d'Asís Lluís Aguiló Gost
Jordi Caelles Abillar

PRIOR SKILLS

Ability to apply the basic tools of differential and integral calculus in one real variable.
Ability to operate with complex numbers, both in binomial form and exponential form.
Ability to factor polynomials over the real and complex numbers.
Ability to decompose rational functions into a sum of partial fractions.
Ability to use the basic tools of linear algebra for the study of systems of linear equations.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:

1. CE1. Ability to solve arithmetic problems related to engineering. Aptitude to apply knowledge concerning: linear algebra, geometry, differential geometry, differential and integral calculus, differential and partial equations, numerical methods, numerical algorithms, statistics and optimization.

Transversal:

2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.
3. EFFECTIVE USE OF INFORMATION RESOURCES - Level 1. Identifying information needs. Using collections, premises and services that are available for designing and executing simple searches that are suited to the topic.

TEACHING METHODOLOGY

In the lectures the instructor presents some motivating ideas, the fundamental concepts and some relevant developments, intermingled with key examples and the resolution of representative problems. In some sessions, one or more problems can be assigned to be individually solved in classroom and to be graded.

In the lab classes the students solve some problems numerically, using MATLAB.

LEARNING OBJECTIVES OF THE SUBJECT

Calculate the Fourier series of a given periodic function, both in real and complex form, and represent and analyze its spectrum.

Understand the role of differential equations in engineering and science, and see some examples.

Compute the solution of the simplest classes of differential equations of first and second order in the time or Laplace domains, and use the appropriate numerical methods when necessary.

Compute and explain some characteristics, quantities and functions related to linear differential equations with constant coefficients, such as stability, characteristic time or transfer functions.

STUDY LOAD

Type	Hours	Percentage
Self study	90,0	60.00
Hours small group	7,5	5.00
Hours large group	52,5	35.00

Total learning time: 150 h

CONTENTS

1 - Fourier series

Description:

How to compute the Fourier series of a periodic function and how to understand it.

- Periodic signals.
- Fourier series in real and complex form.
- Spectrum of a periodic signal. Band-limited signals.
- Comments on the convergence of Fourier series. Gibbs' phenomenon.

Specific objectives:

The student will be able to

- compute the period of a signal, if any, and decide whether the signal is band-limited or not.
- compute the general term of the Fourier series of simple periodic functions, in real and complex form, and write down and simplify the series.
- identify the relative weight of the several harmonics in a periodic function and explain the effect of the jump discontinuities.
- compute, represent and understand the spectrum of a periodic function.
- recognize from the spectrum whether a signal is band-limited or not.

Related activities:

A1, A2, A3, A5

Full-or-part-time: 38h

Theory classes: 12h

Laboratory classes: 2h

Self study : 24h

2 - Differential equations

Description:

Introduction to ordinary differential equations (ODE): what they are, what they are good for, what a solution is. Examples in modelling.

- What is an ordinary differential equation (ODE).
- Solution of an ODE.
- Classification of ODE.
- Solution of ODE with separated variables.
- Problems in mechanics, electricity and population growth models.
- Numerical methods. Euler's method.

Specific objectives:

The student will learn how to

- identify an ODE and some types of ODE.
- check an ODE solution.
- compute the arbitrary constants in a solution, given appropriate initial conditions.
- recognize the ODE in simple problems from mechanics, electricity and population models.
- solve ODE with separated variables.
- explain the existence of different numerical methods for ODE, and the possible errors associated to their use.
- transform an arbitrary ODE to a form suitable for the numerical computation of the solutions.
- use MATLAB's commands and drivers for the numerical solution of ODE and the analysis of the solutions.

Related activities:

A1, A2, A3, A5

Full-or-part-time: 38h

Theory classes: 8h

Laboratory classes: 4h

Self study : 26h

3 - Laplace transform

Description:

How to use the main properties of the Laplace transform to solve linear ODE with constant coefficients.

- Laplace transform of an admissible function. First examples.
- Basic properties of the Laplace transform.
- Tables of Laplace transforms. Inverse Laplace transform.
- Further properties of the Laplace transform.
- Using the Laplace transform to solve linear ODE with constant coefficients.
- ODE with piecewise-defined right-hand sides. The Heaviside step function and the translation-in-time property.
- Systems of linear ODE with constant coefficients. Resolution using the Laplace transform. Characteristic polynomial of a system.

Specific objectives:

The student will be able to

- compute the Laplace transform of a function from the definition in simple cases.
- compute the Laplace transform of a function from known transforms and using the properties of the transform.
- compute inverse Laplace transforms using the properties of the transform and the partial fraction decomposition.
- compute the solution to linear ODE with constant coefficients using the Laplace transform.
- write a piecewise-defined function using the Heaviside function.
- compute the solution to linear constant-coefficient ODE with discontinuous with piecewise-defined right-hand sides.
- compute the solution to systems of linear constant-coefficient ODE using the Laplace transform.

Related activities:

A1, A4, A5

Full-or-part-time: 37h

Theory classes: 13h

Self study : 24h

4 - Analysis of the solutions of linear systems

Description:

How to compute important properties, in the time and Laplace domains, of the solutions of linear constant-coefficient ODE (LCCODE), both for first-order and second-order equations and for first-order systems.

- General solution of a LCCODE.
- Computation of a solution for a LCCODE with sinusoidal right-hand side using phasors.
- Stability. Transient and stationary regimes. Characteristic time.
- Mechanical and electrical examples.
- Stability of LCCODE first-order systems.
- Control systems. Transfer function and impulse response of a linear input/output system.

Specific objectives:

The student will learn how to

- compute the general solution of a LCCODE using the Laplace transform.
- decide the stability of a first-order or second-order LCCODE, and compute and explain the meaning of the characteristic time for the stable case.
- tell the stationary regime from the transient one for a stable LCCODE.
- compute the stationary regime of a LCCODE with sinusoidal right-hand side using phasors.
- decide the stability of first-order systems of LCCODE.
- compute the transfer function and the impulse response of a linear control system.

Related activities:

A1, A4, A5

Full-or-part-time: 37h

Theory classes: 13h

Self study : 24h

GRADING SYSTEM

It is a continuous evaluation grading system, which is implemented through the following activities:

A1=laboratory exam;
A2=exam of parts 1 and 2;
A3=exam of parts 2, 3 and 4;
A4=exam of parts 1, 2, 3 and 4.

The global grade is obtained as follows from the scores (on a 0-10 scale) of the above activities:

FINAL GRADE = MAX (0.2·A1 + 0.3·A2 + 0.5·A3, 0.2·A1 + 0.8·A4).

EXAMINATION RULES.

All activities are individual.

Only activity A4 is re-gradable.

Activities A2, A3 and A4 are written, problem-solving exams. They are re-gradable. Notes, books and digital devices are not permitted.

BIBLIOGRAPHY

Basic:

- Batlle, Carles; Massana, Immaculada; Zaragoza, Marisa. Àlgebra i equacions diferencials [on line]. Barcelona: Edicions UPC, 2000 [Consultation: 22/02/2022]. Available on: <https://upcommons.upc.edu/handle/2099.3/36225>. ISBN 9788483019757.
- Bronson, Richard. Ecuaciones diferenciales [on line]. 3a ed. México: McGraw-Hill, 2008 [Consultation: 16/02/2024]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=4231. ISBN 9789701065099.
- Simmons, George Finlay. Ecuaciones diferenciales: teoría, técnica y práctica [on line]. México D. F: McGrawHill, 2007 [Consultation: 19/02/2024]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=4312. ISBN 9789701061435.

Complementary:

- Boyce, William E. Ecuaciones diferenciales y problemas con valores en la frontera. 5a ed. México etc: Limusa Wiley, 2010. ISBN 9786070501517.
- Ricardo, Henry. Ecuaciones diferenciales: una introducción moderna [on line]. Barcelona: Reverté, 2008 [Consultation: 19/02/2024]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7727. ISBN 9788429151626.
- Antonijuan, Josefina; Batlle, Carles; Boza, Santiago; Prat, Joana D'Arc. Matemàtiques de la telecomunicació [on line]. Barcelona: Edicions UPC, 2001 [Consultation: 04/04/2022]. Available on: <https://upcommons.upc.edu/handle/2099.3/36249>. ISBN 8483015757.

RESOURCES

Other resources:

MATLAB software:
<https://es.mathworks.com/>