

Course guide

200603 - PIPE - Probability and Stochastic Processes

Last modified: 09/06/2023

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| Unit in charge: | School of Mathematics and Statistics | |
| Teaching unit: | 749 - MAT - Department of Mathematics. | |
| Degree: | MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Optional subject). | |
| Academic year: 2023 | ECTS Credits: 5.0 | Languages: English |

LECTURER

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|-------------------------------|---|
| Coordinating lecturer: | JOSE FABREGA CANUDAS |
| Others: | Segon quadrimestre: JOSE FABREGA CANUDAS - A MIGUEL ANGEL FIOL MORA - A |

PRIOR SKILLS

Students should be familiar with the topics covered in a first undergraduate course on probability. In particular, basic knowledge of the following subjects is assumed:

- Elementary probability theory.
- Basic probability models: binomial, geometric, Poisson, uniform, exponential, and normal distributions.
- Random variables. Joint probability distribution and density functions. Independence and correlation.

Concepts necessary to follow the course can be found, for example, in the following references:

- C.M Grinstead and J.L. Snell, Introduction to Probability (chap. 1-7), http://www.dartmouth.edu/chance/teaching_aids/books_articles/probability_book/book
- J. Blitzstein and J. Hwang, "Introduction to Probability", CRC Press, 2019.
- S. Ross, A First Course in Probability, 8th ed., Pearson Education International, 2010.
- M. Sanz-Solé, Probabilitats, Univ. Barcelona, 1999.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:

2. CE-2. Ability to master the proper terminology in a field that is necessary to apply statistical or operations research models and methods to solve real problems.
3. CE-3. Ability to formulate, analyze and validate models applicable to practical problems. Ability to select the method and / or statistical or operations research technique more appropriate to apply this model to the situation or problem.

Transversal:

1. TEAMWORK: Being able to work in an interdisciplinary team, whether as a member or as a leader, with the aim of contributing to projects pragmatically and responsibly and making commitments in view of the resources that are available.

TEACHING METHODOLOGY

Weekly class hours combine both theoretical and practical sessions. The lectures carefully present the fundamental concepts and main results, illustrated with examples. We explain some mathematical proofs, which, for their content and development, are particularly interesting from the learning and creative point of view. The practical sessions discuss the solution to various exercises and problems.

Lists of exercises and guided work could be assigned to be carried out individually or in groups.

LEARNING OBJECTIVES OF THE SUBJECT

The general aim of the course is to introduce the students to the modelling of random phenomena. The course focuses on stochastic convergence problems that are crucial to statistics (laws of large numbers and central limit theorem) and random processes (branching processes, random walks, Markov chains, the Poisson process). Tools related to transform methods (generating and characteristic functions) are also introduced. Special attention is given to the study of specific applications of theoretical concepts.

Skills to be learned:

- Usage of probability and moment generating functions and characteristic functions.
- To know the multivariate normal law and how to operate with jointly Gaussian random variables.
- To understand the different modes of convergence of sequences of random variables and the precise meaning of the laws of large numbers and the central limit theorem.
- Basic concepts on stochastic processes.
- To work with Markov chains and the meaning of stationary distributions and ergodic theorems.
- To understand the Poisson process.
- To identify probability models based on the theoretical results presented in the course.

STUDY LOAD

| Type | Hours | Percentage |
|-------------------|-------|------------|
| Hours small group | 15,0 | 12.00 |
| Self study | 80,0 | 64.00 |
| Hours large group | 30,0 | 24.00 |

Total learning time: 125 h

CONTENTS

1. Generating Functions and Characteristic Function

Description:

- 1.1 Probability and moment generating functions.
- 1.2 The characteristic function.
- 1.3 Sum of a random number of independent random variables.
- 1.4 Distributions with random parameters.
- 1.5 Application to the sample mean and sample variance.

Full-or-part-time: 14h 30m

Theory classes: 3h

Laboratory classes: 1h 30m

Self study : 10h

2. Branching Processes

Description:

- 2.1 The Galton-Watson process.
- 2.2 Application to population growth.
- 2.3 Probability of ultimate extinction.
- 2.4 Probability generating function of the n-th generation.

Full-or-part-time: 11h

Theory classes: 1h 30m

Laboratory classes: 1h 30m

Self study : 8h

3. The Multivariate Gaussian Distribution

Description:

- 3.1 Joint characteristic function of independent Gaussian random variables.
- 3.2 The multidimensional Gaussian law.
- 3.3 Linear transformations.
- 3.4 Lineal dependence and singular Gaussian distributions.
- 3.5 Multidimensional Gaussian density.

Full-or-part-time: 16h

Theory classes: 4h 30m

Laboratory classes: 1h 30m

Self study : 10h

4. Sequences of Random Variables

Description:

- 4.1 The weak law of large numbers. Convergence in probability.
- 4.2 The central limit theorem. Convergence in distribution.
- 4.3 Convergence in mean square.
- 4.4 The strong law of large numbers. Almost-sure convergence.
- 4.5 Borel Cantelli lemmas. Examples of application.
- 4.6 Application to statistical estimation.

Full-or-part-time: 17h 30m

Theory classes: 4h 30m

Laboratory classes: 3h

Self study : 10h

5. Random Walks

Description:

- 5.1 One-dimensional random walks.
- 5.2 Returns to the origin.
- 5.3 Random walks in the plane and the space.
- 5.4 Introduction to brownian motion.

Full-or-part-time: 16h

Theory classes: 4h 30m

Laboratory classes: 1h 30m

Self study : 10h

6. Markov Chains

Description:

- 6.1 Markov chains. The Markov property.
- 6.2 Chapman-Kolmogorov equations.
- 6.3 Recurrent and transient states.
- 6.4 Absorbing chains.
- 6.5 Stationary and limiting distributions.
- 6.6 Application to Montecarlo methods.

Full-or-part-time: 25h

Theory classes: 6h

Laboratory classes: 3h

Self study : 16h

7. The Poisson Process

Description:

- 7.1 Continuous-time stochastic processes: basic concepts.
- 7.2 The Poisson process.
- 7.3 Intertransition times.
- 7.4 Birth and death processes.

Full-or-part-time: 25h

Theory classes: 6h

Laboratory classes: 3h

Self study : 16h

GRADING SYSTEM

The final grade (NF) will be calculated in the following manner:

$$NF = \max(EF, 0.4*EF+0.4*EP+0.2*T)$$

where EF is the final exam mark, EP is the partial exam mark and T is the mark of the exercises and assigned work throughout the course.

BIBLIOGRAPHY

Basic:

- Gut, A. An Intermediate course in probability [on line]. Springer Verlag, 1995 [Consultation: 04/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-0162-0>. ISBN 0387945075.
- Durrett, R. Essentials of stochastic processes [on line]. Springer-Verlag, 1999 [Consultation: 04/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4614-3615-7>. ISBN 038798836X.

Complementary:

- Grimmett, G.R.; Stirzaker, R.R. Probability and random processes. 3rd ed. Oxford Univ. Press, 2001. ISBN 0198572220.
- Sanz Solé, M. Probabilitats. Univ. de Barcelona, 1999. ISBN 8483380919.
- Ross, S.M. Introduction to probability models [on line]. 10th ed. Academic Press, 2010 [Consultation: 04/07/2023]. Available on: <https://www-sciencedirect-com.recursos.biblioteca.upc.edu/book/9780123756862/introduction-to-probability-models>. ISBN 9780123756862.
- Tuckwell, H.C. Elementary applications of probability. 2nd ed. Chapman & Hall, 1995. ISBN 0412576201.
- Grimmett, G; Welsh, D. Probability : an introduction [on line]. Oxford: Oxford University Press, 2014 [Consultation: 04/07/2023]. Available on :



<https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1791152>. ISBN 9780191019920.

- Blitzstein, J.K.; Hwang, J. Introduction to probability. Second edition. CRC Press, Taylor & Francis Group, 2019. ISBN 9781138369917.