Course guide
200901 - SAGDM - Algebra, Geometry and Discrete Mathematics Seminar

Last modified: 23/06/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 3.0 **Languages:** English

LECTURER

Coordinating lecturer: ENRIC VENTURA CAPELL

Others: Segon quadrimestre:
ENRIC VENTURA CAPELL - A

PRIOR SKILLS

The student must know the basics on graph theory. Additionally, it is necessary to have some knowledge on probability theory, group theory and arithmetics.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

TEACHING METHODOLOGY

This seminar will be based on the presentation (from the professors of the course and the students) of material in the context of the theory of expanders. This material will be taken from specialized books, research papers and surveys.

LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the seminar is to show an area of â□□â□□mathematics that intersects both algebra and geometry, discrete mathematics and other related areas, such as computer science, low-dimensional geometry and probability theory, among others.

The main objective is to get the student to gain a basic knowledge of expander theory, as well as the various applications in various branches of contemporary mathematics. In this direction, the student will also be encouraged to learn to conduct technical talks in public and in the preparation of technical scientific documents.

STUDY LOAD

Type	Hours	Percentage
Self study	51,0	68.00
Hours large group	24,0	32.00

Total learning time: 75 h

CONTENTS

Spectral graph Theory and expanders

Description:
- Spectral graph theory.
- Spectral bounds: spectral gap, Mixing Lemma, Alon-Boppana. Ramanujan graphs.
- Existence of graph expanders: Margulis construction. Probabilistic contructions.
- Zig-zag product and construction of expanders.

Full-or-part-time: 20h
Theory classes: 6h 40m
Self study : 13h 20m

Graph expanders and group theory

Description:
- Cayley graphs. Properties
- Random walks in Cayley graphs.

Full-or-part-time: 9h 20m
Theory classes: 6h
Self study : 3h 20m

Graf expanders and Number Theory

Description:
- Sum-product phenomena. Balog-Szemerédi-Gowers Theorem.
- Classical algebraic groups and quasi-random groups.
- Expansion in SL2(Fq): Helfgott Theorem and Bourgain-Gamburd.

Full-or-part-time: 12h
Theory classes: 6h
Self study : 6h

Other applications of graph expanders

Description:
- Applications in knot theory: the Barzdin-Kolmogorov theorem.
- Applications in theoretical computer science: design of concentrators and coding theory.
- Applications in theoretical computing: design of algorithms.
- Analogues in Riemannian geometry.

Full-or-part-time: 12h
Theory classes: 6h
Self study : 6h

GRADING SYSTEM

The grading of this seminar will be based on three points: (CA) Continuous evaluation, (MP) Material preparation and (PT) presentation.

(CA): will be based on the understanding of the material, as well as the meetings that will be held between the student and the responsible to prepare the student's presentation (or presentations). It will also include the fact of being active during the seminar sessions.
(MP): preparation of both the presentation, its good preparation and the summary sheet.
(PT): presentation. This will include questions from the teacher and students.

Overall grading of the seminar: 30% (CA)+20% (MP)+50% (PT)

BIBLIOGRAPHY

Basic:
- Kowalski, Emmanuel. An Introduction to expander graphs. Sócieté Mathématique de France, 2019. ISBN 9782856298985.
- Davidoff,Giuliana; Sarnak, Peter; Valette, Alain. Elementary number theory, group theory and Ramanujan graphs [on line]. London Mathematical Society, 2003 [Consultation: 10/07/2023]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/elementary-number-theory-group-theory-and-ramanujan-graphs/7932F64548F1B38B95AA2593E0B986B2. ISBN 9780521531436.
- Hoory, Shlomo; Linial, Nathan; Wigderson, Avi. "Expander graphs and their applications". Bulletin of the American Mathematical Society [on line]. [Consultation: 10/07/2023]. Available on: https://www.ams.org/journals/bull/2006-43-04/S0273-0979-06-01126-8/S0273-0979-06-01126-8.pdf.- Krebs, Mike; Shaheen, Anthony. Expander families and Cayley graphs : a beginner's guide [on line]. Oxford University Press, 2011 [Consultation: 10/07/2023]. Available on: https://web-p-ebscohost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=2edb9be3-0d08-4c98-8afd-062a20793602%40redis&vid=0&format=EB.
- Ireland, K.; Rosen, M.. A Classical introduction to modern number theory. New York: Springer Verlag, 1990. ISBN 038797329X.

Complementary:
- Lubotzky, Alex. Discrete groups, expanding graphs and invariant measures. Birkhäuser, 1994. ISBN 376435075X.