Course guide
250965 - METNUMAVA - Advanced Discretization Methods

Last modified: 28/03/2024

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN NUMERICAL METHODS IN ENGINEERING (Syllabus 2012). (Optional subject).
ERASMUS MUNDUS MASTER'S DEGREE IN COMPUTATIONAL MECHANICS (Syllabus 2013). (Optional subject).

Academic year: 2023 **ECTS Credits:** 5.0 **Languages:** English

LECTURER

Coordinating lecturer: MATTEO GIACOMINI

Others: PEDRO DIEZ MEJIA, MATTEO GIACOMINI

TEACHING METHODOLOGY

The course consists of 1.3 hours per week of classroom activity (large size group) and 1.3 hours weekly with half the students (medium size group).

The 1.3 hours in the large size groups are devoted to the theoretical lectures in which the topics of the subject are introduced to the students, examples are shown and exercises are solved.

The 1.3 hours in the medium size groups are devoted to solving practical problems with greater interaction with the students. The objective of these practical sessions is to consolidate the general and specific learning objectives.

The remaining weekly hours are devoted to laboratory classes.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENeA: content, learning programme, assessment activities and literature.

LEARNING OBJECTIVES OF THE SUBJECT

Advanced topics of modern numerical
techniques for partial differential equations are presented, with application to a wide variety
of problems in science, engineering, and other fields. Topics include Advanced Finite Elements
(Discontinuous Galerkin, level sets, X-FEM) and mesh-free methods.

* Understand the different theoretical and computational aspects of a wide spectrum of methods. * Develop skills for the practical application of diffenert methods and implementation problems associated to each one of them. * Emphasis will be put on the need for students to acquire independence in their studies; theyhave to learn to use a computer for basic programming and learn to use and make the most of their study hours. * Implement and use computer programs to solve non-linear problems on different fields of application. * To analyse from a critical point of view the results obtained by the simulations.

Advanced Finite Elements:
* Discontinuous Galerkin (DG) for hiperbolic problems. Riemann solvers and numerical
fluxes.
* DG for elliptic operators.
* Extended finite elements (X-FEM) and applications (crack simulation, holes and
inclusions, material interfaces).
* Level sets.

Mesh-free Methods:
* Overview of mesh-free methods.
* Moving least squares approximation.
* Element-free Galerkin method.
* Smooth particle hydrodynamics.
* Implementation of essential boundary conditions.
* Coupling of finite elements and mesh-free methods.
* Particle finite element methods.
* Discrete element methods.
* Overview of method and applications.
* Basic formulation.

This modules covers a selection of advanced topics on the numerical approximation of partial differential equations, with application to a variety of problems in science and engineering, including wave propagation, multiphase flows, free surface flows, fracture mechanics, nonlinear elasticity and phase transition.

Learning objectives:
To be able to understand the fundamentals of advanced discretisation techniques for partial differential equations beyond the classical finite element method (FEM), starting from specific engineering applications for which traditional FEM approaches show numerical limitations. Particular emphasis will be given to the numerical properties of the methods, computational advantages and disadvantages of each approach and main implementation details. The specific objectives of the module are to be able to:
- formulate accurate discretisations using high-order methods (review of continuous and discontinuous Galerkin methods);
- understand the concept of geometry approximation errors and the different strategies to describe geometry in a FEM framework (high-order meshes, exact geometry, immersed boundaries, level-set);
- understand the notion of partition of unity (generalised FEM);
- formulate a FEM solver using an unfitted mesh (cut FEM);
- introduce appropriate enrichment strategies in GFEM (eXtended FEM);
- describe interface phenomena (phase-field models);
- understand the rationale of particle methods (meshless methods, smooth-particle hydrodynamics).

STUDY LOAD

Type	Hours	Percentage
Hours medium group	9,8	7.83
Hours small group	9,8	7.83
Self study	80,0	63.95

Type	Hours	Percentage
Hours large group	25,5	20.38

Total learning time: 125.1 h

CONTENTS

Review of FEM & high-order methods

Description:
CG, strong and weak imposition of Dirichlet BC, DG, high-order polynomials, static condensation, hybridisation, efficient implementation, matrix-free methods
Exercises on this topic
Computer lab session on this topic and written assignment to be submitted

Specific objectives:
To formulate accurate discretisations using high-order methods

Full-or-part-time: 14h 23m
Theory classes: 2h
Practical classes: 2h
Laboratory classes: 2h
Self study : 8h 23m

Geometry description

Description:
Polynomial approximation of the geometry, mesh generation, high-order meshes, exact geometry, NURBS, immersed surfaces, level-set, Hamilton-Jacobi equation
Exercises on this topic
Computer lab session on this topic and written assignment to be submitted

Specific objectives:
To understand the concept of geometry approximation errors and the different strategies to describe geometry in a FEM framework

Full-or-part-time: 13h 12m
Theory classes: 2h
Practical classes: 2h
Laboratory classes: 1h 30m
Self study : 7h 42m

Partition of unity methods

Description:
Partition of unity methods, generalised FEM, cut FEM, integration on cut elements, badly cut elements, ill-conditioning issues (agglomeration, extrapolation, ...), XFEM, enrichment strategies
Exercises on this topic
Computer lab session on this topic and written assignment to be submitted

Specific objectives:
To understand the notion of partition of unity, to formulate a FEM solver using an unfitted mesh, to introduce appropriate enrichment strategies in generalised FEM

Full-or-part-time: 36h
Theory classes: 6h
Practical classes: 6h
Laboratory classes: 3h
Self study : 21h

Phase-field models

Description:
Physical description of phase transition models, phase-field models, Stefan equation, Allen-Cahn equation
Exercises on this topic
Computer lab on this topic and written assignment to be submitted

Specific objectives:
To describe interface phenomena

Full-or-part-time: 20h 24m
Theory classes: 3h 30m
Practical classes: 3h 30m
Laboratory classes: 1h 30m
Self study : 11h 54m

Particle methods

Description:
Meshless methods, smooth-particle hydrodynamics, fast dynamics
Exercises on this topic
Computer lab session on this topic and written assignment to be submitted

Specific objectives:
To understand the rationale of particle methods

Full-or-part-time: 24h
Theory classes: 4h
Practical classes: 4h
Laboratory classes: 2h
Self study : 14h

GRADING SYSTEM

The grade of the course is obtained from a continuous assessment during the module. This consists of several activities, both individual and in group, of incremental training, carried out during the module, both in and out of the classroom.

The final grade will be computed as follows:
- 35% written exam on the first part of the module (Test 1);
- 35% written exam on the second part of the module (Test 2);
- 30% classwork (Assignments on practical and programming exercises).

The written tests will assess the assimilation of the fundamental concepts related to the learning objectives of the module and will consist of:
- theoretical questions on the numerical methods presented in class;
- practical exercises requiring to write the discrete formulation for a given method and problem;
- interpretation questions commenting on the expected performance of the methods starting from the theory.

The evaluation of the classwork will assess the incremental learning of the students and will be based upon:
- assignments consisting of both written and programming exercises on the numerical methods seen during the module, to be submitted for correction;
- participation during lectures, exercise and practical classes.

For the distance-learning version of the Master, classwork evaluation will only consider the submitted assignments.

EXAMINATION RULES.

The assignments must be submitted via ATENeA by the announced deadline. Late submissions or assignments submitted using other means will not be accepted and will be graded 0.

The assignments must be performed individually: students are encouraged to discuss about the assignments but the submitted work must be the result of one own efforts. Plagiarism in the assignments will be punished with a 0 in the classwork grade.

The written exams (tests 1 and 2) must be performed individually and will be closed-book. Plagiarism will be punished with a 0 in the module grade.

BIBLIOGRAPHY

Basic:
- Moës, Nicolas; Dolbow, John; Belytschko, Ted. A finite element method for crack growth without remeshing. Wiley Online Library, 1999.
- Provatas, N.; Elder, K. Phase-field methods in materials science and engineering [on line]. Weinheim: Wiley-VCH Verlag GmbH & Co. KGaA, 2010 [Consultation: 04/02/2021]. Available on: https://onlinelibrary.wiley.com/doi/book/10.1002/9783527631520. ISBN 9783527407477.
- Huerta, A., Belytschko, T., Fernández-Méndez, S., Rabczuk, T., Zhuang, X., and Arroyo, M.. Meshfree Methods. 2a. Wiley Online Library, 2017. ISBN 9781119176817.

Complementary:
- Huerta, A.; Angeloski, A.; Roca, X. and Peraire, J.. Efficiency of high-order elements for continuous and discontinuous Galerkin methods. Wiley, 2013.
- Sevilla, R.; Fernandez-Mendez, S. and Huerta, A.. Comparison of high-order curved finite elements. Wiley, 2011.
- Sevilla, R.; Fernández-Méndez, S. and Huerta, A.. NURBS-Enhanced Finite Element Method (NEFEM). A Seamless Bridge Between CAD and FEM. Springer, 2011.
- Hospital-Bravo, R.; Sarrate, J. and Díez, P.. Numerical modeling of undersea acoustics using a partition of unity method with plane waves enrichment. Springer, 2016.
- Burman, E.; Claus, S.; Hansbo, P.; Larson, M.G.; Massing, A.. CutFEM: Discretizing geometry and partial differential equations. Wiley, 2015.