UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Course guide
250COD001 - 250COD001 - Continuum Mechanics

Last modified: 16/07/2026

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN COMPUTATIONAL AND DATA-ASSISTED ENGINEERING (Syllabus 2026). (Compulsory subject).

Academic year: 2026 **ECTS Credits:** 5.0 **Languages:** English

LECTURER

Coordinating lecturer: Oriol Lloberas Valls

Others: Oriol Lloberas Valls
Javier Bonet

PRIOR SKILLS

The student must have completed the basic core subjects of a degree course in engineering. In concrete terms, basic knowledge of algebra, calculus, differential geometry, mechanics will be necessary.

REQUIREMENTS

The student must have completed the basic core subjects of a degree course in engineering. In concrete terms, basic knowledge of algebra, calculus, differential geometry, mechanics will be necessary.

TEACHING METHODOLOGY

The classes include theory, problems and guided activities. For each subject, the necessary theoretical concepts are first introduced, then some examples and exercises are solved by the professor. The students will solve some additional problems during the class time under the supervision of the professor. On the other hand, students will solve some additional assignments as homework.

LEARNING OBJECTIVES OF THE SUBJECT

This is a complete course in nonlinear continuum mechanics for engineers. It carries out a deep review of the fundamental concepts, including motion, deformations, strains, stresses, conservation laws of momentum, energy and entropy, and an introduction to the Mechanics of solids and fluids.

* The students will be able to understand and assimilate the foundations of the solid and fluid mechanics, identifying the most important aspects of material modeling, and dissipation mechanisms associated with nonlinear behaviour.
* They have to be able to interpret the physical meaning of the material properties and properly identify the differential equations for the solution of problems of continuum mechanics, with its application to the elastic behaviour of solids, and learn the foundations of fluid mechanics.
* The students will develop practical skills to work with tensors and formulate and develop analysis of diverse problems of solids and fluids in engineering.

STUDY LOAD

Type	Hours	Percentage
Self study	80,0	64.00
Hours large group	45,0	36.00

Total learning time: 125 h

CONTENTS

Mathematical preliminaries

Description:
• Vector algebra: Scalar product, vector components, indicial notation, cross product, triple product.
• Tensor algebra: Second order tensors, tensor product of two vectors, tensor components, indicial notation, matrix notation, dot product, transpose, trace, determinant, symmetric tensors, eigenvalues and eigenvectors, higher order tensors.
• Directional derivative: definition, properties, examples of linearization, function minimisation.
• Vector and tensor analysis: Gradient operator, divergence, curl, Laplacian, integration theorems.
• Problems & Exercises

Specific objectives:
The objectives are to introduce the tensor notation to be used in the course and to familiarize students with the use of tensor notation.

Full-or-part-time: 5h
Theory classes: 5h

Kinematics

Description:
This topic describes the main assumptions of the Mechanics of Continuous Media and the main concepts of non-linear kinematics of particles are introduced: the mathematical description of motion, velocity, acceleration, trajectories, material and spatial coordinates, change of volume, strain tensors and small displacements and strains.

Contents:
• Motion: initial and current configurations; Equations of motion; particle trajectories; Lagrangian/material coordinates; Eulerian/spatial coordinates.
• Deformation gradient: fibre map, area map and volume map, push forward, pull back, polar decomposition.
• Deformation: strain measures, Cauchy-Green tensors; Green & Almansi strain tensors, linearized strain tensor, small displacements.
• Kinematics: velocity vector, velocity gradient, material and spatial time derivatives, steady states, rate of deformation tensor, rate of change of volume.
• Problems & Exercises

Specific objectives:
The aim is to introduce the main concepts of nonlinear kinematics of a continuum

Full-or-part-time: 8h
Theory classes: 8h

Conservation Laws I

Description:
This chapter introduces the concept of a conservation laws with associated fluxes and source terms, both in integral and differential forms. The chapter also presents the interaction of the motion and conservation laws, introducing concepts such as Lagrangian, updated Lagrangian and Eulerian versions of conservation laws.

Contents:
• General definition: Conservation variables and control volume; concentration; source terms; surface fluxes; flux vector; master law conservation of vector variables; flux tensor.
• Local conservation laws: differential equations; convective and conductive fluxes; discontinuous solutions -shocks; jump conditions across a shock.
• Conservation laws and the motion: Lagrangian and updated Lagrangian versions of conservation laws; Convective form; Reynolds transport theorem; Eulerian conservation laws.
• Conservation of mass: Material density and Lagrangian conservation of mass; spatial density and continuity equation; Eulerian and Lagrangian jump conditions.
• Geometric conservation laws: Conservation of deformation gradient; conservation of volume.
• Problems & Exercises

Full-or-part-time: 8h
Theory classes: 8h

Conservation Laws II

Description:
This topic applies the general concepts of conservation laws to momentum, energy and entropy to arrive at the general balance laws and the laws of thermodynamics.

Contents:
• Conservation of momentum: linear momentum; external forces; traction vector in Lagrangian and updated Lagrangian form.
• Stress tensors: Cauchy stresses; first Piola-Kirchhoff stress tensor; second Piola-Kirchhoff tensor
• Equilibrium equation: total Lagrangian equilibrium; updated Lagrangian equilibrium equation; Eulerian equilibrium equation; jump conditions.
• Conservation of angular momentum: Symmetry of the Cauchy stresses; symmetry conditions in terms of Piola-Kirchoff stress tensors.
• Principle of virtual work: total Lagrangian and updated Lagrangian versions; variational formulations; work conjugacy; stress-strain conjugate pairs.
• Conservation of Energy: energy sources and fluxes, Lagrangian, updated Lagrangian and Eulerian versions; jump conditions; the first law.
• Entropy and the second law: definition; the second law of thermodynamics; energy dissipation; Fourier law; reversible processes.
• Problems & Exercises

Specific objectives:
The main objectives are to introduce the key balance laws in continuum mechanics, familiarizing students with concepts such as stress tensor, energy and entropy.

Full-or-part-time: 8h
Theory classes: 8h

Constitutive Models for Solids

Description:
This topic describes the classical constitutive equations for solids, ranging from linear elasticity to dissipative models with internal state variables.
Contents:
• Linear elasticity: Linear stress-strain relationships; elasticity tensor; strain energy; dynamic equilibrium equation; boundary and initial conditions; static equilibrium; total potential energy; 2-dimensional solutions, plane stress and plane strain.
• Hyperelasticity: St. Venant model; compressible neo-Hookean model; tangent elasticity tensors; nearly incompressible neo-Hookean model.
• Thermoelasticity: Calorimetry; entropy-temperature relationships; rigid conductor equations; stress evaluation in thermoelasticity; Helmholtz free energy; thermo-mechanical coupling; tangent thermoelasticity tensors.
• Dissipative models: dissipation through viscosity; internal state variables; isotropic damage; evolution of internal state variables.
• Problems & Exercises

Specific objectives:
The main objectives are to introduce linear elasticity, hyperelasticity and thermoelasticity as typical as fundamental constitutive models commonly used for solids.

Full-or-part-time: 8h
Theory classes: 8h

Constitutive Models for Fluids

Description:
This topic describes the classical constitutive equations for fluids, ranging from Newtonian flows, ideal gases and compressible Euler equations.

Contents:
• Newtonian flows: General conservation equations in Eulerian form; Cauchy stresses; Newtonian viscosity model; thermodynamic pressure; general Navier-Stokes equations in conservation and convection forms.
• Incompressible flows: Incompressible Navier-Stokes equations; Stokes flow; potential flows; Bernoulli equation.
• Ideal gas: the ideal gas equation of estate; inviscid compressible Euler equations; acoustic waves; sound speed.
• Problems & Exercises

Full-or-part-time: 8h
Theory classes: 8h

GRADING SYSTEM

The course grade is obtained from the ratings of the continuous assessment (30%) and final exam (70%).
Continuous assessment: The student has to solve along the course and supervised by the professor, several exercises and problems, both in the classroom (during school hours), and beyond.
Final exam: The final exam consists of some questions and problems similar to those that have been raised and resolved in class.

EXAMINATION RULES.

Continuous assessment: Failure to perform a continuous assessment activity in the scheduled dates will result in a zero mark in that activity.
Final exam: The final exam will be a written exam.

BIBLIOGRAPHY

Basic:

- Bonet, J.; Gil, A.J.; Wood R.D. Nonlinear Solid Mechanics for Finite element Analysis : statics. Cambridge: Cambridge University Press, 2016. ISBN 9781107115798.
- Bonet, J.; Wood, R.D. Nonlinear continuum mechanics for finite element analysis [on line]. 2nd Edition. Cambridge: Cambridge University Press, 2008 [Consultation: 28/07/2026]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/nonlinear-continuum-mechanics-for-finite-element-analysis/67AD6DBAAB77E755C09E7FB82565DA0B. ISBN 9780521838702.
- Bonet, J.; Gil A.J.; Wood R,D. Nonlinear Solid Mechanics for Finite Element Analysis : dynamics [on line]. New York: Cambridge University Press, 2021 [Consultation: 28/07/2026]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/nonlinear-solid-mechanics-for-finite-element-analysis-dynamics/1424900AC005178702008378835A7A18. ISBN 9781107115620.