UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Course guide
250COD005 - 250COD005 - Partial Differential Equations and Finite Element Method

Last modified: 16/06/2026

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN COMPUTATIONAL AND DATA-ASSISTED ENGINEERING (Syllabus 2026). (Compulsory subject).

Academic year: 2026 **ECTS Credits:** 5.0 **Languages:** English

LECTURER

Coordinating lecturer: Fernandez Mendez, Sonia

Others: Fernandez Mendez, Sonia
Saez Viñas, Pablo

PRIOR SKILLS

Differential Calculus in several dimensions, and basic knowledge on differential equations.

TEACHING METHODOLOGY

Theoretical sessions with slides, blackboard explanations and exercises, together with computer sessions for the implementation of the FEM method, solution of problems and analysis of results.
Python codes are be provided by the lecturers, and minor modifications or additional implementation are required from the students.

LEARNING OBJECTIVES OF THE SUBJECT

Getting familiar with the mathematical modeling with Partial Differential Equations, and the solution of the corresponding boundary value problems with the Finite Element Method (FEM); focusing on problems modelled by the Laplace equation (heat equation, flow in porous media, potential flow) and its transient version (transient heat equation). Acquiring basic expertise on the derivation and analysis of FEM formulations, their implementation and application, with emphasis on the critical analysis of results.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	36.00
Self study	80,0	64.00

Total learning time: 125 h

CONTENTS

Mathematical modeling with PDE

Description:
Modelling of heat problems: derivation of the transient 3D heat equation, boundary and initial conditions, other applications (flow in porous media, pollutant diffusion, etc), problems with steady solution, problems that can be reduced to 1D or 2D, interface conditions.
Analytical solutions: verification of a tentative solution, superposition principle, separation of variables.

Full-or-part-time: 9h
Theory classes: 9h

Introduction to the Finite Element Method (FEM)

Description:
Finite Element approximation spaces: mesh (nodes, elements), C0 polynomial piece-wise approximations, interpolation properties (delta Kronecker property, convergence) and need for a weak form.
Weak form of the Poisson equation with Dirichlet and Neumann boundary conditions, through weighted residuals.
Discretization of the weak form with a FE approximation and resulting system of equations. Energy minimization and Rayleigh-Ritz method.
Implementation: elemental matrices and assembly, isoparametric transformation and numerical integration, implementation of Dirichlet boundary conditions.
Lax Milgram theorem, Cea's lemma, a priori error bounds, and convergence rates.
Brief overview of numerical methods for the solution of large, sparse system of equations.
Transient heat equation: weak form, discretization in space and system of ODEs, solution with Euler (lumped mass matrix), backward Euler and Crank-Nicolson (trapezoidal rule), stability conditions from the matrix eigenvalues, space and time convergence, and verification of accuracy. Implementation and numerical experiments. Nodal equation in 1D and alternative derivation from finite differences.

Full-or-part-time: 18h
Theory classes: 18h

Mathematical modeling with PDE II

Description:
Wave equation (acoustics, membrane) and possible reduction to 1D. Particular analytical solution. Harmonic solutions and problem statement in frequency domain (eigenvalue problem).
Mathematical and physical classification of PDEs: parabolic (transient heat equation), elliptic (steady heat equation) and hyperbolic (wave equation). Boundary conditions depending on the classification. Hyperbolic equations: characteristic lines, inflow and outflow boundaries.

Full-or-part-time: 6h
Theory classes: 6h

FEM limitations and notes on advanced discretization methods

Description:
FEM limitations: convection dominated problems, moving boundaries and interfaces, high-order PDEs.
Notes on immersed boundary methods, X-FEM, phase-field.
Nitsche's method.

Full-or-part-time: 6h
Theory classes: 6h

GRADING SYSTEM

30% continuous assessment (exercises to be delivered) + 70% exams (two exams)

EXAMINATION RULES.

If not otherwise stated, exams are closed-book.

BIBLIOGRAPHY

Basic:

- Yehuda, P.; Rubinstein, J. Introduction to Partial Differential Equations. Cambridge [etc.]: Cambridge University Press, 2005. ISBN 9780521848862.
- Strauss, Walter A. Partial Differential Equations: An Introduction [on line]. 2nd ed. Hoboken: John Wiley & Sons, 2008 [Consultation: 16/06/2026]. Available on: https://math24.wordpress.com/wp-content/uploads/2016/03/partial-differential-equations-by-w-a-s.pdf. ISBN 9780470054567.
- Quarteroni, Alfio. Numerical Models for Differential Problems [on line]. 3rd ed. Cham: Springer, 2017 [Consultation: 23/06/2026]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-319-49316-9. ISBN 3-319-49316-7.

Complementary:

- Oñate, E.; Diez, P.; Zarate, J.F.; Larese, A. Introduction to the finite element method [on line]. Barcelona: UPC, 2008 [Consultation: 23/06/2026]. Available on: https://hdl.handle.net/2117/184007.
- Zienkiewicz, O.C. The Finite element method: its basis and fundamentals [on line]. 8th ed. Cham: Elsevier Science & Technology, 2024 [Consultation: 29/07/2026]. Available on: https://www-sciencedirect-com.recursos.biblioteca.upc.edu/book/monograph/9780443160448/the-finite-element-method. ISBN 0443160457.