Course guide
250COD008 - 250COD008 - Optimisation

Last modified: 20/07/2026

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN COMPUTATIONAL AND DATA-ASSISTED ENGINEERING (Syllabus 2026). (Compulsory subject).

Academic year: 2026 **ECTS Credits:** 5.0 **Languages:** English

LECTURER

Coordinating lecturer: Rossi Bernecoli, Riccardo

Others: Sibuet Ruiz, Nicolás
Zorrilla Martínez, Rubén

PRIOR SKILLS

master level in mathematics, basic knowledge of programming

TEACHING METHODOLOGY

All classes will contain both theoretical material and practical - computer based - assignements to be solved interactively by the students

LEARNING OBJECTIVES OF THE SUBJECT

teach the fundamentals of mathematical optimization, particularly in view of application to engineering problems and numerical methods. The course will also delivers the fundamental mathematics at the basis of modern Machine Learning.

STUDY LOAD

Type	Hours	Percentage
Self study	80,0	64.00
Hours large group	45,0	36.00

Total learning time: 125 h

CONTENTS

Mathematical Optimization Course

Description:
Part I — Optimization Foundations
Lecture 1 — Optimization Problems and Mathematical Preliminaries
Theory
Examples of applications (shape and topology optimization, sizing, etc)
Structure of optimization problems
Objective functions and constraints

Norms and inner products
Gradients and Hessians
Quadratic approximations
Convex vs non-convex problems
Practice
Visualizing objective landscapes
Numerical gradients and Hessians in NumPy

Lecture 2 — Linear Programming
Theory
Linear programs in standard form
Geometry of polyhedra
Extreme points and optimality
Fundamental theorem of LP
Simplex algorithm (conceptual overview)
Interior-point idea
Practice
Modeling LP problems
Solving LPs with scipy.optimize.linprog

Lecture 3 — Quadratic Programming
Theory
Quadratic forms
Positive definite matrices
Convex quadratic programs
KKT system for equality constraints
Active-set intuition
Practice
Portfolio optimization with CVXPY

Lecture 4 — Applications in Computational Mechanics
Practice – Application to model problems

Part II — Convex Optimization
Lecture 5 — Convex Sets and Convex Functions
Theory
Convex sets
Convex hull and epigraph
Convex functions
First-order convexity condition
Jensen's inequality
Examples from machine learning losses
Practice
Verifying convexity analytically and numerically

Lecture 6 — Lagrangian Duality and KKT Conditions
Theory
General constrained optimization
Lagrangian formulation
Dual function
Karush–Kuhn–Tucker conditions
Optimality interpretation
Practice
Solving nonlinear constrained problems

Lecture 7 — Duality and Sensitivity Analysis
Theory
Lagrangian of linear programs
Dual problem
Weak and strong duality

Complementary slackness
Interpretation of dual variables
Practice
Solving primal and dual LPs in Python
Verifying complementary slackness

Lecture 8 — Applications in Computational Mechanics
Practice
Practical applications hands-on

Part III — Gradient-Based Optimization
Lecture 9 — Gradient Descent
Theory
Steepest descent
Lipschitz gradients
Convergence guarantees
Step size selection
Conditioning
Practice
Implementing gradient descent
Convergence experiments

Lecture 11 — Newton and Quasi-Newton Methods + (Brief intro to) Krylov Methods
Theory
Newton’s method
Quadratic convergence
Hessian approximation
BFGS and quasi-Newton methods
Linear systems in optimization
Krylov subspaces
Conjugate gradient method
GMRES algorithm
Preconditioning
Practice
Comparing Newton vs gradient descent

Part V — Optimization in Machine Learning
Lecture 12 — Stochastic Gradient Methods
Theory
Stochastic gradient descent
Mini-batch optimization
Variance of stochastic gradients
Learning rate schedules
Momentum methods
Adaptive Algorithms (Adam, Adagrad)
Limitations of adaptive optimizers
Practice
Training a neural network using SGD

Lecture 13 — Non-Convex Optimization in Deep Learning
Theory
Landscape of neural network loss functions
Saddle points and flat minima
Overparameterization
Practical training heuristics
Practice
Training a small network and analyzing optimization behavior

Specific objectives:
The specific objective of the course is to enable the students to
- understand the therminology used in mathematical optimization
- cast a given problem into a form suitable to employ mathematical optimization techniques
- understand the characteristics of the problem so to choose the "best" optimization technique

Related activities:
the course will propose several "toy problems" of increasing complexity, to verify how the different techniques perform in finding "optimal" solutions.
some of such problems will be solved interactively in the class, others will be assigned as tasks and be part of the final evaluation

Full-or-part-time: 45h
Theory classes: 45h

GRADING SYSTEM

50% homeworks
50% theory

EXAMINATION RULES.

tasks will be assigned during the group with an associated deadline. students will have to resolve the tasks (alone or in groups) and prepare a presentation of the work done, to be evaluated in a oral exam

BIBLIOGRAPHY

Basic:
- Boyd, Stephen P.; Vandenberghe, Lieven. Convex Optimization [on line]. Cambridge: Cambridge University Press, 2004 [Consultation: 28/07/2026]. Available on: https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf. ISBN 0521833787.

RESOURCES

Other resources:
https://web.stanford.edu/~boyd/cvxbook/