UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Course guide
34952 - AG - Algebraic Geometry

Last modified: 29/05/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2025 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer:

Others:

PRIOR SKILLS

Aquaintance with mathematical computations, both by hand and with a computer, and mathematical reasoning, including proofs.

REQUIREMENTS

Basic abstract Algebra, Topology and Differential Geometry.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Approximately 50% of class time will be dedicated to interactive master classes, in which the lecturer will discuss course topics and propose small challenges and questions to solve. The other half of the class time will be structured as a problem-solving session, in which students will solve problems from a proposed list on the blackboard, based on the course syllabus, and their solutions will be discussed by the class.

LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the course is to introduce students to local algebraic geometry, with a focus on plane curve singularities. It aims to provide insight into the singularity theory of plane curves and the geometric theory of valuations of the ring of convergent series of two variables over the complex numbers. The course will demonstrate that singular points of algebraic curves in the complex plane is a meeting point for various areas of mathematics.

The course will heavily rely on examples, emphasizing the geometric significance of the subject. The specific topics for the final projects will be determined based on the students' interests.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Chapter 1: Algebraic and analytic varieties

Description:
Algebraic preliminaries. Affine algebraic varieties, Nullstellensatz. Analytic varieties, Teorema de Rückert.

Full-or-part-time: 11h 30m
Theory classes: 5h
Self study : 6h 30m

Chapter 2: Parametrizing branches of plane curves

Description:
Newton-Puiseux algorithm, Weierstrass preparation and division theorems, Hensel's lemma. Intersection multiplicity: Halphen's formula. Testing analytic reducibility.

Full-or-part-time: 12h 30m
Theory classes: 6h
Self study : 6h 30m

Chapter 3: Infinitely near points and resolutions of singularities

Description:
Proximity, Enriques diagrams, dual graph. Resolutions of singularities. Rings in successive neighbourhoods. The order of singularity.

Full-or-part-time: 23h
Theory classes: 10h
Self study : 13h

Chapter 4: Topological classification of singularities

Description:
Topological classification of plane curves: equisingularity, semigroup of values, complete equisingularity invariants. Milnor number. Topological conic structure of a singularity. Topology of the singularity link. The Milnor fibration.

Full-or-part-time: 23h
Theory classes: 10h
Self study : 13h

Chapter 5: Constructions on the resolution tree

Description:
Homology of a blow-up. The exceptional divisor of a curve. Functions on the resolution tree. Complete ideals. Multiplier ideals and jumping numbers. Topological zeta function.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 6: Analytic classification of plane curves

Description:
Jacobian ideal and its semimodule of values. Tjurina algebra and Tjurina number. Unfoldings of equations. Deformations of curves: versal and miniversal deformations. Zariski's moduli space. Teissier's monomial curve.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 7: Valuations and complete ideals

Description:
Classification of valuations. Zariski decomposition of complete ideals.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 8: Final projects

Description:
The final essays of the course on the topics chosen by the students will be presented by the students themselves and commented by the course lecturers.

Full-or-part-time: 25h
Theory classes: 5h
Self study : 20h

GRADING SYSTEM

Students who solve a sufficient number of problems on the blackboard during the problem-solving class will pass the course. If they wish to improve their grade from a passing grade to a higher score, they will be assigned a final project. The final project will involve studying, writing an essay and delivering a lecture on an additional topic towards the end of the course.

Students who have not actively participated enough in the problem-solving class, or still wish to improve their grade even after completing the problem class and final project, will be required to take a final exam lasting approximately 4 hours.

EXAMINATION RULES.

The problem list for participation in the problem-solving class will be published at the beginning of each course unit. Students are expected to prepare these problems in advance at home.

The topics for optional final projects aimed at increasing grades will be proposed around Easter. Students will be responsible for preparing the lecture and the essay of the final project independently at home.

Students who choose to take the final exam will be required to do so without any notes, books, or other materials whatsoever.

BIBLIOGRAPHY

Basic:
- Casas Alvero, Eduardo. Singularities of plane curves. Cambridge: Cambridge University Press, 2000. ISBN 0521789591.
- Fischer, Gerd. Plane algebraic curves. American Mathematical Society, cop. 2001. ISBN 0821821229.
- Ghys, E. A Singular mathematical promenade. Lyon: CNRS-ENS Éditions, 2017. ISBN 9782847889390.
- Wall, C. T. C. Singular points of plane curves. Cambridge, UK ; New York: Cambridge University Press, cop. 2004. ISBN 0521547741.
- Zariski, Oscar; Linchtin, Ben; Teissier, Bernard. The Moduli problem for plane branches. American Mathematical Society, 2006. ISBN 9780821829837.

Complementary:
- Brieskorn, Egbert; Knörrer, Horst. Plane algebraic curves : translated by John Stillvell [on line]. Springerlink, 2012 [Consultation: 15/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-0348-0493-6. ISBN 9783034804929.
- Casas-Alvero, Eduardo. Algebraic curves, the brill and noether way. Springer, ISBN 9783030290153.
- Chenciner, Alain. Courbes algébriques planes [on line]. Berlin: Springer, 2008 [Consultation: 07/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-33708-9. ISBN 9783540337072.
- Greuel, Gert-Martin; Lossen, Christoph; Shustin, Eugenii. Singular algebraic curves : with an appendix by Oleg Viro. Springer, ISBN 9783030033491.
- Kollar, János. Lectures on resolution of singularities. Princeton University Press, ISBN 9780691129235.