

## Course guide

### 34953 - NT - Number Theory

Last modified: 01/06/2023

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| <b>Unit in charge:</b>     | School of Mathematics and Statistics   |                           |
| <b>Teaching unit:</b>      | 749 - MAT - Department of Mathematics.   |                           |
| <b>Degree:</b>             | MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).<br>(Optional subject). |                           |
| <b>Academic year:</b> 2023 | <b>ECTS Credits:</b> 7.5   | <b>Languages:</b> English |

#### LECTURER

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| <b>Coordinating lecturer:</b> | JOAN CARLES LARIO LOYO   |
| <b>Others:</b>                | Primer quadrimestre:<br>JOAN CARLES LARIO LOYO - A<br>BERNAT PLANS BERENGUER - A |

#### PRIOR SKILLS

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Basic knowledge of algebraic structures: groups, rings and fields.

#### REQUIREMENTS

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Basic material covered in any standard courses on arithmetic, group theory and Galois theory.

#### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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##### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

##### Transversal:

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

#### TEACHING METHODOLOGY

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Most of the lectures will take place on the blackboard (replaced by online lessons if necessary), explaining carefully the contents of the course and providing as much explicit examples, exercises and applications as possible. The students will be encouraged to consult suitable references and to discuss between them and with the professor in order to achieve a good understanding of the material.

## LEARNING OBJECTIVES OF THE SUBJECT

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- 1) Algebraic number theory.
- 2) Number theory in function fields.
- 2) Cyclotomic theory.

## STUDY LOAD

| Type              | Hours | Percentage |
|-------------------|-------|------------|
| Self study        | 127,5 | 68.00      |
| Hours large group | 60,0  | 32.00      |

**Total learning time:** 187.5 h

## CONTENTS

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### Algebraic Number Theory

**Description:**

Introduction  
Cyclotomic extensions  
Cyclotomic polynomials

**Full-or-part-time:** 10h

Theory classes: 10h

### Number Theory in function fields

**Description:**

Functions fields over finite fields  
Carlitz polynomials  
Carlitz extensions

**Full-or-part-time:** 10h

Theory classes: 10h



## Cyclotomic Theory

### Description:

Cyclotomic integers  
Cyclotomic units  
Unique factorization  
Class numbers  
Galois action  
Kronecker-Weber theorem  
Regular polygons  
Fermat equation  
Quadratic reciprocity

Carlitz modules  
Galois action  
Carlitz-Hayes theorem  
Cyclotomic and Carlitz analogies  
Quadratic reciprocity  
Drinfeld modules

**Full-or-part-time:** 40h

Theory classes: 40h

## GRADING SYSTEM

There will be a final exam.

## EXAMINATION RULES.

Solved exercises and works must be delivered according to schedule.

## BIBLIOGRAPHY

### Basic:

- Marcus, D. A. Number fields [on line]. 2a. Springer, 2018 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=6315071>. ISBN 9783319902326.
- Washington, Lawrence C. Introduction to cyclotomic fields. 2a. Springer, 1997. ISBN 9780387947624.
- Samuel, Pierre. Algebraic theory of numbers [on line]. Dover, 2008 [Consultation: 10/07/2023]. Available on: <https://web-s-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=df04de0b-9ea0-463f-837a-8c3e0eeaf5c3%40redis&vid=0&format=EK>. ISBN 9780486466668.
- Lang, Serge. Cyclotomic fields I and II. 2a. Springer, 1990. ISBN 9780387966717.
- Weiss, Edwin. Algebraic number theory [on line]. Dover, 1998 [Consultation: 10/07/2023]. Available on: <https://web-s-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=41dffc72-5826-4437-902d-0e8cf20a0103%40redis&vid=0&format=EK>. ISBN 9780486154367.
- Goss, David. Basic structures of function field arithmetic. Springer, 1998. ISBN 9783540635413.
- Rosen, Michael. Number theory in function fields [on line]. Springer, 2002 [Consultation: 10/07/2023]. Available on: <https://web-s-ebshost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=c3a1ddbe-02a6-4928-9cf8-61fa67b451e8%40redis&vid=0&format=EB>. ISBN 9780387953359.



## RESOURCES

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### Computer material:

- SAGE. Mathematical Software
- Matlab. Mathematical software