Course guide
34957 - GT - Graph Theory

Last modified: 08/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: GUILLEM PERARNAU LLOBET

Others: Primer quadrimestre:
GUILLEM PERARNAU LLOBET - A
ORIOL SERRA ALBO - A

PRIOR SKILLS

Elementary Calculus and Linear Algebra; basic notions and skills in combinatorics and probability.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Combination of theoretical lectures and exercise classes, with student presenting their solutions to the proposed problems. The active participation of students is a requirement for the course assessment.

LEARNING OBJECTIVES OF THE SUBJECT

Basics of Graph Theory.
Random graphs.
Applications of random graphs: the probabilistic method.
Spectral techniques to the study of graphs.
Applications of spectral techniques: expansion and random walks.
Extremal graph theory
Applications of extremal techniques: graph limits.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

An introduction to Graph Theory

Description:
Introduction to random graphs, main properties and classical theorems.

Specific objectives:
Basic terminology and notation
Paths and cycles
Distance and Diameter
Connectivity
Trees
Matchings
Colorings

Related competencies :
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
06 URI. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
05 TEQ. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
03 TLG. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
04 COE. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
07 AAT. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Full-or-part-time: 12h
Theory classes: 12h

Random graphs

Description:
Introduction to classical model of random graphs and its main combinatorial properties.

Specific objectives:
Erdos-Rényi model of random graphs.
Properties of almost all graphs.
First and second moment methods.
The probabilistic method.
Threshold functions.
Method of Moments
Chernoff's inequality
Component phase transitions

Related competencies :
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
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Full-or-part-time: 12h
Theory classes: 12h

Spectral Graph Theory

Description:
Introduction to spectral graph theory, graph expanders and applications to random walks on graphs.

Specific objectives:
Adjacency and Laplacian matrices. Courant-Fischer Theorem. Interlacing. Cospectral graphs. Graph invariants: independence number, chromatic number, diameter,... Spectral gap and second largest eigenvalue. Isoperimetry and Cheeger's inequality. Matrix tree theorem. Shannon capacity. Random walks in graphs.

Related competencies :
MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
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Full-or-part-time: 16h
Theory classes: 16h

Extremal Graph Theory

Description:
Introduction to Extremal Graph Theory, the Szemeredi Regularity Lemma, and applications to Graph limits

Specific objectives:
Mantel Theorem, Turán Theorem, Erdos-Stone-Simonovits Theorem, Szemeredi Regularity Lemma, Counting Lemma, Triangle Removal Lemma, Graph Limits

Full-or-part-time: 20h
Theory classes: 20h

GRADING SYSTEM

The assessment of the course is as follows:
- weekly work on the proposed problems and their presentation during the lectures, 30% of the mark
- a final comprehensive exam on the course topics, 70% of the mark

EXAMINATION RULES.

The active participation in the course is a requirement for the final assessment.

BIBLIOGRAPHY

Basic:
- Alon, Noga; Spencer, Joel. The Probabilistic method. 2016. Wiley, 2015. ISBN 9781119061953.
- Brouwer, A. E; Haemers, W. H. Spectra of graphs [on line]. New York [etc.]: Springer, cop. 2012 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4614-1939-6. ISBN 9781461419389.
- Diestel, Reinhard. Graph theory. 3rd ed. Berlin: Springer, 2005. ISBN 3540261826.
- Hell, Pavol; Nesetril, Jaroslav. Graphs and homomorphisms. Oxford: Oxford University Press, 2004. ISBN 0198528175.
- Frieze, Alan; Karonski, Michal. Introduction to random graphs [on line]. Cambridge, [etc.]: Cambridge University Press, cop. 2016 [Consultation: 10/07/2023]. Available on: https://www-cambridge-org.recursos.biblioteca.upc.edu/core/books/introduction-to-random-graphs/0F67A19795B731B0C97EAB5BB5748CF2. ISBN 9781107118508.

Complementary:
- Biggs, Norman L. Algebraic graph theory. 2nd ed. New York: Cambridge University Press, 1993. ISBN 0521458978.
- Chung, Fan R. K. Spectral graph theory. Providence: American Mathematical Society, 1997. ISBN 0821803158.
- Lovász, László. Large networks and graph limits. Providence, R.I.: American Mathematical Society, cop. 2012. ISBN 9780821890851.