

## Course guide

# 34963 - ACPDE - Advanced Course in Partial Differential Equations

Last modified: 01/06/2023

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.  
981 - CRM - Mathematical Research Centre.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2023    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** OLLI SAARI

**Others:** Segon quadrimestre:  
OLLI SAARI - A

### PRIOR SKILLS

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Basic knowledge of Partial Differential Equations (undergraduate level).  
Basic knowledge of Mathematical Analysis (undergraduate level).  
Basic knowledge of Functional Analysis (undergraduate level).

### REQUIREMENTS

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Undergraduate courses in Partial Differential Equations and in Mathematical Analysis.  
It is helpful an undergraduate course in Functional Analysis.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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#### Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

#### Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

## TEACHING METHODOLOGY

Classes will combine theoretical aspects and proofs with resolution of concrete problems and exercises.  
Further reading from the bibliography will be given often.

## LEARNING OBJECTIVES OF THE SUBJECT

This course is intended to be an introduction to modern methods for solving elliptic partial differential equations. However, some insights to classical solutions to parabolic and hyperbolic equations will also be given. The objectives of the course are:

- understand the classical methods to solve the transport, wave, heat, Laplace, and Poisson equations,
- understand the role of Sobolev norms and compact embeddings to solve PDE and find spectral decompositions,
- learn the modern methods to solve elliptic PDE.

## STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

**Total learning time:** 187.5 h

## CONTENTS

### Classical methods in PDEs

#### Description:

[This topic will only be treated in the exercises sessions.] Classical solutions to the transport, wave, heat, Laplace, and Poisson equations. Maximum principles, Green's functions, separation of variables, energy methods, probabilistic interpretation.

**Full-or-part-time:** 46h

Theory classes: 15h

Self study : 31h

### Hilbert space techniques

#### Description:

Orthogonal projections, Riesz-Fréchet representation theorem, Lax-Milgram theorem.

**Full-or-part-time:** 25h

Theory classes: 8h

Self study : 17h

### Sobolev spaces

#### Description:

Mollifiers, Fréchet-Kolmogorov theorem, distributions, Sobolev norms, Poincaré inequality, compact embeddings, approximation by smooth functions, traces.

**Full-or-part-time:** 29h

Theory classes: 9h

Self study : 20h

### Weak formulation and the weak maximum principle

**Description:**

Weak solutions via Hilbert space techniques and interpretation, comparison principles in the weak formulation.

**Full-or-part-time:** 25h

Theory classes: 8h

Self study : 17h

### Eigenvalues

**Description:**

Spectral decompositions, applications to (time dependent) evolution equations, Rayleigh quotient, description of the first eigenvalue for the Dirichlet problem on a bounded domain.

**Full-or-part-time:** 17h

Theory classes: 6h

Self study : 11h

### Regularity theory

**Description:**

Boundedness of weak solutions, Sobolev-Gagliardo-Nirenberg inequality, regularity in Sobolev spaces, the translation method, bootstrap technique.

**Full-or-part-time:** 25h 30m

Theory classes: 8h

Self study : 17h 30m

### Nonlinear problems

**Description:**

Calculus of variations, monotone iteration method, obstacle problems.

**Full-or-part-time:** 20h

Theory classes: 6h

Self study : 14h

## GRADING SYSTEM

The evaluation of the course is based on:

- the resolution of problems proposed in class (40%),
- a midterm exam (20%),
- a final comprehensive exam (40%).

The active participation during the course will be a requirement for the evaluation of the final exam.

## BIBLIOGRAPHY

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### Basic:

- Brézis, H. Análisis funcional : teoría y aplicaciones. Madrid: Alianza, 1984. ISBN 8420680885.
- Chipot, M. Elliptic equations : an introductory course [on line]. Basel: Birkhäuser Basel, 2009 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-7643-9982-5>. ISBN 9783764399825.
- Evans, Lawrence C. Partial differential equations. Providence, Rhode Island: American Mathematical Society, 1998. ISBN 0821807722.
- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan: Springer, 2008 [Consultation: 10/07/2023]. Available on: <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3062992>. ISBN 9788847007512.

### Complementary:

- Struwe, Michael. Variational methods : applications to nonlinear partial differential equations and hamiltonian systems [on line]. 2nd rev. and substantially expanded ed. Berlin: Springer, 1996 [Consultation: 10/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-74013-1>. ISBN 3540520228.
- Folland, G. B. Introduction to partial differential equations. 2nd ed. Princeton (N.J.): Princeton University Press, 1995. ISBN 0691043612.
- Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. 2nd ed., rev. third printing. Berlin: Springer-Verlag, 1998. ISBN 354013025X.
- Necas, Jindrich. Introduction to the theory of nonlinear elliptic equations. Chichester: John Wiley & Sons, 1986. ISBN 0471908940.