200004 - CD - Differential Calculus

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2019
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Compulsory)
ECTS credits: 7,5  Teaching languages: Catalan, Spanish

Teaching staff

Coordinator: ENRIC VENTURA CAPELL

Others: Segon quadrimestre:
  MARINA GARROTE LOPEZ - M-A, M-B
  LAZARIO ALBERTO LARRAURI BORROTO - M-A, M-B
  ÓSCAR RIVERO SALGADO - M-B
  NATALIA SADOVSKAIA NURIMANOVA - M-A
  ENRIC VENTURA CAPELL - M-A, M-B

Degree competences to which the subject contributes

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the
   constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

General:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically
   associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often
   within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or
   unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or
   limited information, but that include reflecting on social and ethical responsibilities linked to the application of their
   knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these
   objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of
    this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best
    action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-
    appraisal. Choosing the best path for broadening one's knowledge.
## Learning objectives of the subject

(Section not available)

## Study load

<table>
<thead>
<tr>
<th>Total learning time: 187h 30m</th>
<th>Hours large group:</th>
<th>45h</th>
<th>24.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hours medium group:</td>
<td>0h</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>Hours small group:</td>
<td>30h</td>
<td>16.00%</td>
</tr>
<tr>
<td></td>
<td>Guided activities:</td>
<td>7h 30m</td>
<td>4.00%</td>
</tr>
<tr>
<td></td>
<td>Self study:</td>
<td>105h</td>
<td>56.00%</td>
</tr>
</tbody>
</table>

### Teaching methodology

(Section not available)
### Content

| 1. Topology of \( \mathbb{R}^n \). Sequences of vectors.                                                                 | Learning time: 25h |
| - Euclidean, normed and metric spaces. Case study: \( \mathbb{R}^n \).                                                                 | Theory classes: 6h |
| - Open and closed sets. Interior, exterior and boundary of a set.                                                      | Practical classes: 4h |
| - Sequences in \( \mathbb{R}^n \). Limit. Cauchy sequences and completeness. Characterization of closed sets by sequences. | Self study : 15h   |
| - Connected sets.                                                                                                       | |

**Description:**
- Functions of several variables. Level sets and graphics of real functions
- Limit of a function at a point (special emphasis on the case of two variables).
- Continuity at a point and a set. Properties of continuous functions.
- Continuity and compactness. Weierstrass theorem.
- Uniform continuity. Heine-Cantor theorem.
- Equivalence norms and equivalence metrics. Fixed point theorem.

| 2. Limits and continuity of functions.                                                                                   | Learning time: 25h |
| - Continuity and compactness. Weierstrass theorem.                                                                        | Theory classes: 6h |
| - Uniform continuity. Heine-Cantor theorem.                                                                               | Practical classes: 4h |
| - Equivalence norms and equivalence metrics. Fixed point theorem.                                                         | Self study : 15h   |

**Description:**
- Functions of several variables. Level sets and graphics of real functions
- Limit of a function at a point (special emphasis on the case of two variables).
- Continuity at a point and a set. Properties of continuous functions.
- Continuity and compactness. Weierstrass theorem.
- Uniform continuity. Heine-Cantor theorem.
- Equivalence norms and equivalence metrics. Fixed point theorem.

| 3. Differentiability.                                                                                                     | Learning time: 34h |
| - Differentiability at a point. Hyperplane tangent to the graph of a real function.                                    | Theory classes: 8h |
| - Differentiability and operations. Chain rule. relationship between differentiability, continuity and partial derivatives. | Self study : 20h   |
| - Differentiability in an open set. Mean Value Theorem. Functions of class \( C^1 \).                                  | |
| - Differentiable curves. Tangent vector.                                                                                 | |

**Description:**
- Differentiability at a point. Hyperplane tangent to the graph of a real function.
- Differentiability and operations. Chain rule. relationship between differentiability, continuity and partial derivatives.
- Differentiability in an open set. Mean Value Theorem. Functions of class \( C^1 \).
- Differentiable curves. Tangent vector.
4. Theorems of differentiable functions.  

**Learning time:** 41h  
Theory classes: 10h  
Practical classes: 6h  
Self study: 25h

**Description:**  
- The inverse function theorem. Diffeomorphisms.  
- The implicit function theorem. Derivatives of implicit functions.  
- Rank theorems.

5. Taylor formula. Local extrema.  

**Learning time:** 33h  
Theory classes: 8h  
Practical classes: 5h  
Self study: 20h

**Description:**  
- Taylor formula. Expressions of the rest.  
- Local extrema. Critical points.  
- Classification of critical points: quadratic forms, Hessian matrix.  
- Criteria of Silvester and of eigenvalues of the Hessian matrix.

6. Submanifolds of R^n and constrained extrema.  

**Learning time:** 32h  
Theory classes: 7h  
Practical classes: 5h  
Self study: 20h

**Description:**  
- Submanifolds of R^n. Tangent vectors. Tangent and normal spaces at a point.  
- Parameterized and implicit submanifolds. Regular curves and surfaces.  
- Constrained extrema and Lagrange multipliers.  
- Absolute extrema.

**Qualification system**

Final Mark = Max(Final Exam, 0.7*Final Exam + 0.3*Midterm Exam)  
Eventually, the grading of the mid-term exam could be modified by other grades.

An extra exam will take place on July for students that failed during the regular semester.
Bibliography

Basic:


Complementary:


Others resources: