200102 - AR - Real Analysis

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2018
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Compulsory)
ECTS credits: 7.5
Teaching languages: Catalan

Teaching staff

Coordinator: JAIME FRANCH BULLICH
Others: Segon quadrimestre:
  JAIME FRANCH BULLICH - A, B
  PAU MARTIN DE LA TORRE - A, B

Prior skills

Knowledge in Differential and Integral Calculus in one and several variables.

Degree competences to which the subject contributes

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

General:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
The course has to be for the student a transition between Calculus and Mathematical Analysis. Because of that, an important goal for the student has to be to become used to the utility of abstraction and conceptual methods. Even though the abstract and conceptual character is the most important, the calculus aspects of some parts (Fourier series, integrals depending of one parameter) have to be fully reached.

The course has to be useful as a preparation for the use of Mathematical Analysis in other courses like Ordinary Differential Equations (where uniform convergence is more used), Partial Differential Equations (where the mean square convergence is more used) and Functional Analysis (where the knowledge on function spaces is further developed). It can also be useful as a preparation for postgraduate courses on subjects like signal analysis or function theory.

### Study load

<table>
<thead>
<tr>
<th>Total learning time: 187h 30m</th>
<th>Hours large group: 45h 24.00%</th>
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</thead>
<tbody>
<tr>
<td>Hours medium group: 0h 0.00%</td>
<td>Hours small group: 30h 16.00%</td>
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<tr>
<td>Guided activities: 0h 0.00%</td>
<td>Self study: 112h 30m 60.00%</td>
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</tbody>
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Theory classes will consist of expositions by the teacher of the definitions, the statements, the proofs and the examples. In problem classes the exercises from a given list will be solved, but the expositions could be shared between teachers and some students. Some homework exercises will also be assigned. Among the goals of the course the ability to solve problems will be considered more important than the simple acquisition of knowledge, and because of that intuition and creativity will be strongly promoted. This relative importance will also affect the evaluation procedures.
### Content

<table>
<thead>
<tr>
<th>Topic</th>
<th>Learning time: 48h 30m</th>
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<tbody>
<tr>
<td><strong>Topology in the space of continuous functions.</strong></td>
<td>Theory classes: 12h</td>
</tr>
<tr>
<td></td>
<td>Practical classes: 8h</td>
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<td></td>
<td>Self study: 28h 30m</td>
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**Description:**
- Sequences and series of functions: pointwise and uniform convergence.
- Stone-Weierstrass Theorem.
- Equicontinuous families.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Learning time: 62h 30m</th>
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<tbody>
<tr>
<td><strong>Lebesgue measure and integration in R^n.</strong></td>
<td>Theory classes: 15h</td>
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<td></td>
<td>Practical classes: 10h</td>
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<tr>
<td></td>
<td>Self study: 37h 30m</td>
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**Description:**
- Measurable sets and measurable functions.
- Integration of measurable functions.
- Dominated convergence.
- Fubini's Theorem.
- Integral calculus and integrals depending on parameters.
- Lp spaces.

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<thead>
<tr>
<th>Topic</th>
<th>Learning time: 48h 30m</th>
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<tbody>
<tr>
<td><strong>Fourier series.</strong></td>
<td>Theory classes: 12h</td>
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<tr>
<td></td>
<td>Practical classes: 8h</td>
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<tr>
<td></td>
<td>Self study: 28h 30m</td>
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**Description:**
- Fourier series in L2.
- Fourier series of periodic functions.
- Pointwise and uniform convergence.

### Qualification system

There will be two marks (over 10 points): the midterm exam mark (P) and the final exam mark (F). The midterm exam mark could be increased due to the exercises assigned to students as a homework. The final mark of the subject is the maximum between F and (0.3·P+0.7·F).

An extra exam will take place on July for students that failed during the regular semester.
Bibliography

Basic:


Complementary: